The Open University of Sri Lanka
B.Sc/ B.Ed Degree Programme
Open Book Test (OBT) -2023/2024
Pure Mathematics – Level 05
PEU5305 – Complex Analysis I

DURATION: ONE HOUR

Date: 30.07.2023 **Time: 1.00 p.m. – 2.00 p.m.**

ANSWER **ALL** QUESTIONS.

1. Let $f(z) = \begin{cases} \frac{z\,\mathrm{Re}(z)\mathrm{Im}(z)}{|z|^2} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$ be a complex valued function.

   i. Show that $f(z)$ satisfies Cauchy-Riemann equations at $z = 0$.

   ii. Is $f(z)$ differentiable at $z = 0$? Justify your answer.

2.

   i. Express $\frac{1}{z+1}$ in a power series centered at 1.

   ii. Find the radius of convergence of the power series obtained in part (i).

3. Prove that $\tan z$ is analytic except at $z = (2n+1)\frac{\pi}{2}$, where n is an integer, by proving that $\frac{d}{dz}(\tan z) = \sec^2 z$ for $z \neq (2n+1)\frac{\pi}{2}$, where n is an integer.

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