

The Open University of Sri Lanka

B.Sc./B.Ed. Degree/Continuing Education Programme – Level 04

Open Book Test (OBT) – 2023/2024

Applied Mathematics

ADU4302/ADE4302- Vector Calculus

Duration :- One Hour



Date :- 21-07-2023

Time :- 1.00 p.m. – 2.00 p.m.

Answer All Questions.

1. (a) Find the domain and range of the function  $f(x, y) = \sqrt{x^2 + y^2 - 4}$ .

(b) Sketch the level curves of the function  $f(x, y) = x^2 + y^2 - 2x - 2y$ .

2. Evaluate the following limits, if they exist:

(a)  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$

(b)  $\lim_{(x,y) \rightarrow (0,0)} \frac{4xy^2}{x^2 + 3y^4}$

3. If  $z = xy + f(x^2 + y^2)$  then show that  $y \frac{\partial z}{\partial x} - x \frac{\partial z}{\partial y} = y^2 - x^2$ .

4. Using chain rule, find  $\frac{df}{dt}$  in terms of  $t$ , for the function  $f(x, y) = 4x^2 + 3y^2$ , where  $x = \sin t$  and  $y = \cos t$ .

5. Find Taylor's expansion of  $f(x, y) = e^{x+y}$  about the point  $(0, 0)$ .

6. Find the directional derivative of the function  $f(x, y) = y^2 \sin 2x$  at the point  $P\left(\frac{\pi}{4}, 2\right)$  in the direction of  $\underline{u} = 5\underline{i} + 12\underline{j}$ .

7. Find the equation of the tangent plane to the surface  $z = 4 - x^2 - y^2$  at the point  $P(1, 1, 2)$ .

8. Find the local extrema of the function  $f(x, y) = x^3 + 2xy - 6x - 4y^2$  and determine their nature.

