

THE OPEN UNIVERSITY OF SRI LANKA

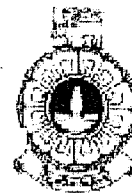
B.Sc. /B.Ed. Degree Programme

APPLIED MATHEMATICS-LEVEL 05

APU3141/APE5141- Linear Programming

Final Examination 2015/2016

Duration: Two Hours



**Date: 28.06.2016** **Time: 01.00 p.m- 03.00 p.m**

Answer four questions only

(1) A diet conscious housewife to ensure certain minimum intake of vitamin *A*, *B* and *C* for the family. The minimum daily (quantity) needs of the vitamins *A*, *B*, *C* for the family are respectively 30, 20 and 16 units. For the supply of these minimum vitamin requirements, the housewife relies on two fresh foods. The first one provides 7, 5, 2 units of the three vitamin per gram respectively and the second one provide 2, 4, 8 units of the same three vitamins per gram of the food stuff respectively. The first food stuff costs Rs. 30 per gram and second Rs.20 per gram. The problem of how many grams of each food stuff the housewife buy everyday to keep her food bill as low as possible.

- (i) Formulate the problem as a Linear Programming problem.
- (ii) Write the Dual problem.
- (iii) Solve the Dual problem by using Simplex method.
- (iv) Solve the primal problem graphically.

(2) The Sri Lankan navy wishes to assign 4 ships *A*, *B*, *C*, *D* to patrol 4 sectors (1, 2, 3, 4) of the sea. In some areas ships are to be on the lookout for illegal fishing boats, and in other sectors to watch for enemy submarines, so the commander rates each ship in terms of its probable efficiency in each sector. These relative efficiencies are given in the table below:

Sector/ Ship	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
1	20	60	50	55
2	60	30	80	75
3	80	100	90	80
4	65	80	75	70

On the basis of the rating shown, the commander wants to determine the patrols assignments producing the greatest over all efficiencies. What is the optimal assignment?

(3) A departmental store wishes to purchase the following quantities of ladies' dresses:

Dress type	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
Quantity	150	100	75	250

Tenders are submitted by three different manufacturers who undertake to supply not more than the quantities below:

Manufacturer	<i>X</i>	<i>Y</i>	<i>Z</i>
Total Quantity	350	250	150

The store estimates that its profit per dress will vary with the manufacturer as shown in the matrix below.

<b>Dress manufacturer</b>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
<b>X</b>	2	4	5	2
<b>Y</b>	3	4	5	1
<b>Z</b>	2	3	4	2

- (i) Formulate a linear programming model to determine the maximum profit.
  - (ii) Find the initial solution using Vogel's Approximation method in order to maximize the profit.
  - (iii) Find the optimal solution.
- (4) The tableau below is the initial tableau for a linear programming problem in  $X_1$ ,  $X_2$  and  $X_3$ . The objective is to maximize the profit,  $Z$ .

<b>Basis</b>	$X_1$	$X_2$	$X_3$	$s_1$	$s_2$	$s_3$	<b>Solution</b>
$s_1$	12	4	5	1	0	0	246
$s_2$	9	6	3	0	1	0	153
$s_3$	5	2	-2	0	0	1	171
<b>Z</b>	-2	-4	-3	0	0	0	0

Using the information in the tableau, write down

- (i) the objective function,
- (ii) the three constraints as inequalities with integer coefficients.
- (iii) Solve this linear programming problem.

(5) Consider the following Primal problem:

$$\text{Minimize } z = 3y_1 + 5y_2,$$

$$\text{Subject to } 2y_1 \leq -4,$$

$$3y_2 \leq -6,$$

$$3y_1 + 2y_2 \leq -18,$$

$$y_1, y_2 \geq 0.$$

- (i) Write down the dual problem for the above primal problem.
- (ii) Solve the dual problem given in (i) by using the dual simplex method. Hence, write the solution of the primal problem.
- (6) (i) Briefly explain the role of artificial variables in linear programming.
- (ii) What are the artificial variable techniques used in linear programming?
- (iii) Use Big-M method to solve the following linear programming problem:

$$\text{Minimize } z = 2x_1 + x_2,$$

$$\text{Subject to } 3x_1 + x_2 = 3,$$

$$4x_1 + 3x_2 \geq 6,$$

$$x_1 + 2x_2 \leq 3,$$

$$x_1, x_2 \geq 0.$$

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