The Open University of Sri Lanka
B.Sc. / B.Ed. Degree Programme – Level 05
Final Examination -2015/2016
Applied Mathematics
AMU3183— Numerical Methods

Duration: Two Hours

Date: 03. 07. 2016 **Time: 01.00 p.m. – 03.00 p.m.**

Answer Four Questions Only.

1. (a) Prove that

(i) $E = \Delta + 1$, (ii) $E = (1-\nabla)^{-1}$, (iii) $\delta = E^{1/2} - E^{-1/2}$,

(iv) $\nabla\Delta = \Delta\nabla = \delta^2$ (v) $\Delta - \nabla = \delta^2$

where Δ, ∇, δ and E are the forward difference, the backward difference, the central difference and the shift operators respectively.

(b) Derive the Gregory- Newton forward interpolation formula.

(c) Hence, interpolate $f(45)$ given that $f(x)$ passes through the points (40, 31), (50, 73), (60, 124), (70, 159) and (80, 190).

2. (a) (i) Derive the Newton's general interpolation formula with divided differences.

(ii) Hence, find the value of $f(8)$, given that $f(x)$ passes through the points

(4, 48), (5, 100), (7, 294), (10, 900), (11, 1210) and (13, 2028).

(b) (i) Derive the Lagrange's interpolation formula.

(ii) Applying Lagrange's formula inversely, obtain the root of the equation $f(x) = 0$, given that $f(30) = -30$, $f(34) = -13$, $f(38) = 3$ and $f(42) = 18$.

3. (a) Derive Simpson's One Third Rule.

(b) If the interval $[a, b]$ is divided into $2n$ sub intervals then show that the error in Simpson's one third rule is given by $|E| < \frac{(b-a)h^4}{180} M$ where M is the numerically greater value of $y_0^{iv}, y_2^{iv}, \cdots y_{2n-2}^{iv}$.

(c) Applying Simpson's One Third rule for following data,

sin0	$\sin(\pi/12)$	$\sin(\pi/6)$	$\sin(\pi/4)$	$\sin(\pi/3)$	$\sin(5\pi/12)$	$\sin(\pi/2)$
0	0.2588	0.5000	0.7071	0.8660	0.9659	1.0

evaluate the integral $\int_0^{\pi/2} \sin x \, dx$.

4. (a) (i) Derive formula for the Euler's method to solve $\frac{dy}{dx} = f(x, y)$ subject to the initial condition $y(x_0) = y_0$.

(ii) Solve $\frac{dy}{dx} = 3x^2 + 1$ with the initial condition $y(1) = 2$ using the Euler's method. Estimate $y(2)$ taking $h = 0.25$.

(b) (i) Derive formula for the modified Euler method to solve $\frac{dy}{dx} = f(x, y)$ subject to the initial condition $y(x_0) = y_0$.

(ii) Using modified Euler method solve $\frac{dy}{dx} = 3e^x + 2y$ with the initial condition $y(0) = 0$.

Estimate $y(1)$ taking $h = 0.25$

5. (a) Applying Taylor series method of fourth order for the differential equation

$\frac{dy}{dx} = xy^2 + 1$ subject to the initial condition $y(0) = 1$, evaluate $y(0.2)$ and $y(0.4)$.

(b) Applying Taylor series method of fourth order for the system of differential equations $\frac{dy}{dx} = x + z$ and $\frac{dz}{dx} = x - y^2$ subject to the initial conditions $y(0) = 2$ and $z(0) = 1$, evaluate $y(0.1)$, $y(0.2)$, $z(0.1)$ and $z(0.2)$.

6. (a) State fourth order Runge-Kutta algorithm to solve $\frac{dy}{dx} = f(x, y)$ subject to the initial condition $y(x_0) = y_0$.

(b) Solve $\frac{dy}{dx} = 1 + y^2$ with the initial condition $y(0) = 0$ using Runge-Kutta method of fourth order. Evaluate the value of y, when $x = 0.2$ and $x = 0.4$.

(c) Solve $\frac{d^2y}{dx^2} = y^3$ with the initial condition $y(0) = 10$, $y'(0) = 5$ using Runge-Kutta method of fourth order and evaluate $y(0.1)$.

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