The Open University of Sri Lanka
Department of Mathematics
B.Sc/ B.Ed Degree Programme
Final Examination - 2016/ 2017
Applied Mathematics– Level 05
APU3244 - Graph Theory

DURATION: - THREE HOURS

Date: 15 – 01 – 2018 **Time: 1.30 p.m. – 4.30 p.m.**

ANSWER FIVE QUESTIONS ONLY

01. (a) Write down the number of *edges* in each of the following generalized standard graphs:

(i) K_n (ii) $K_{m,n}$ (iii) C_n (iv) W_n (v) Q_n

(b) By drawing a graph as an illustrative example, show that each of the following statements is *true*:

(i) The length of the longest simple *circuit* in K_5 is 10,

(ii) A connected graph need not have a *Hamiltonian path*,

(iii) A *non-Eulerian* graph can be *orientable*,

(iv) If a graph is *Eulerian*, then its *line graph* is both *Eulerian* and *Hamiltonian*,

(v) The complement of the *bipartite graph* $K_{m,n}$ is the union of the complete graphs K_m and K_n.

(c) Prove each of the following statements:

(i) If a graph G is *bipartite* then each cycle of G has even length,

(ii) If a connected graph is *Eulerian* then the degree of each vertex of the graph is even.

02. (a) Determine whether the following graphs are *color critical* or not. Justify your answer.

(i) C_4 (ii) K_5 (iii) $K_{3,3}$ (iv) $K_{1,5}$

(b) State the *Vizing theorem* and find a *minimal edge coloring* of each of the following generalized standard graphs:

(i) C_n (ii) K_n (iii) $K_{1,n}$ (iv) W_n

(c) How many different channels are needed for six television stations (A, B, C, D, E, F) whose distances (in kilometers) from each other are shown in the following table? Assume that two stations cannot use the same channel when they are within 140 kilometers from each other.

Station	*A*	*B*	*C*	*D*	*E*	*F*
A	--	85	175	100	50	100
B	85	--	125	175	100	130
C	175	125	--	100	200	250
D	100	175	100	--	210	220
E	50	100	200	210	--	100
F	100	130	250	220	100	--

03. (a) Let $E=\{1,2,3,4,5,6\}$ be a set of six elements. Let $S_1=\{1,2,3\}, S_2=\{2,3\}, S_3=\{1,2\}$, $S_4=\{1,3\}$ and $S_5=\{2,5,6\}$ be five subsets of E.

Determine whether $\Im=(S_1, S_2, S_3, S_4, S_5)$ has *transversal* or not. Justify your answer.

(b) Write down the *incidence matrix* A of the family $\Im$. Hence,

(i) find the *term rank*,

(ii) verify the *Konig- Egervacy theorem* for the *incidence matrix* A,

(iii) verify your result obtained in part (a).

(c) Nine school girls walk each day in 3 groups of 3. Obtain a *geometrical construction* to arrange girls walk for 4 days so that in that time, each pair of girls walks together in a group just once.

Hence, write down the girls walk for 4 days.

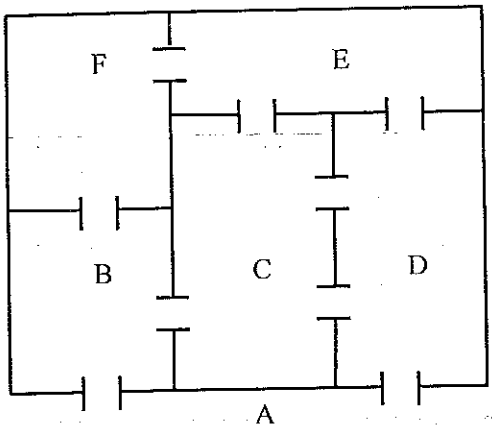
04. (a) Define the number of *lines* of a line graph $L(G)$ of G with p lines and q points each has degree d_i , where $1 \le i \le p$.

(i) Construct the *line graph* $L(K_4)$ of K_4. Is $L(K_4)$ planar? Justify your answer.

(ii) Construct the *total graph* $T(K_3)$ of K_3.

(iii) Show that $L(K_4)$ and $T(K_3)$ are *isomorphic.*

(b) The following map shows the floor plan of a building. Determine whether it is possible to plan a walk that passes through each door exactly once and returns to its starting point A.



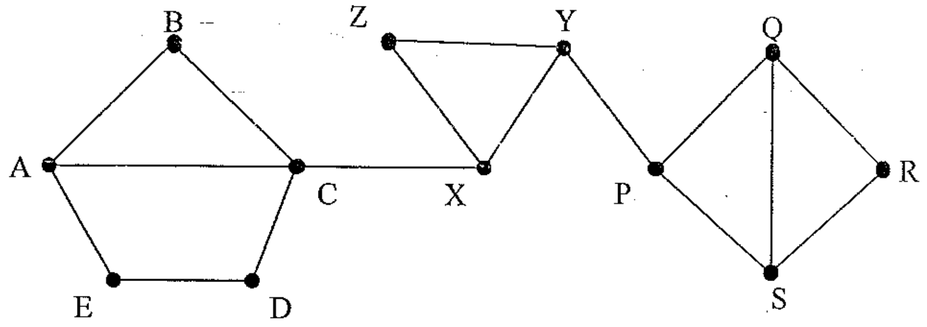
05. (a) Define a *critical planar* graph and find the *crossing number* of each of the graph in **part (a) of the Question no 02.**
Which of them are *critical planar?* Give reasons for your answer.

(b) By briefly explaining the relevant elementary subdivision, show that a sub graph of the *Petersen graph* is *homeomorphic* to $K_{3,3}$.

(c) State the *Euler formula* for planar connected graph and hence determine whether it is possible to draw a connected simple graph with 6 regions and 10 vertices, each of degree 3.

(d) Find the number of edges of the complement of a simple planar graph which has n vertices and m edges.

06. (a) Let G be the following graph.



(i) Find all the *cut points* and all the *blocks* of G,

(ii) Draw the *cut point graph* and the *block graph,*

(iii) How many *bridges* are there in G? Justify your answer.

(b) A building contractor advertised for a bricklayer, a carpenter, a plumber and a toolmaker. He received five applicants in which one for the job of bricklayer, one for carpenter, one for bricklayer and plumber, and two for plumber and toolmaker.

(i) Draw the corresponding bipartite graph.

(ii) Show that the marriage condition does not hold for this problem.

(iii) Can all four jobs be filled by the applicants? Justify your answer.

07. (a) Let $M=(E, I)$ be a *matroid* defined in terms of its independent sets. Let A be a subset of a non-empty finite set E. Define the *rank of* A.

Let $E=\{a, b, c, d, e\}$ be a set of five elements. Find *matroids* on E and write down their standard names in each of the following cases:

(i) E is the only base,

(ii) the empty set is the only base,

(iii) the bases are the subsets of E containing exactly three elements.

(b) Let $E = \{a, b, c, d, e, f, g\}$ be a set of seven elements.

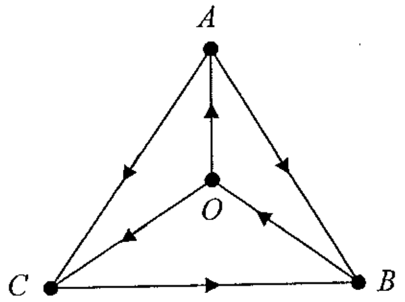
Let $B = \{bcd, bef, bga, ceg, cfa, dea, dfg\}$ be a *family* of 3-element subsets of E.

Draw a *Fano matroid* defined on the set E.

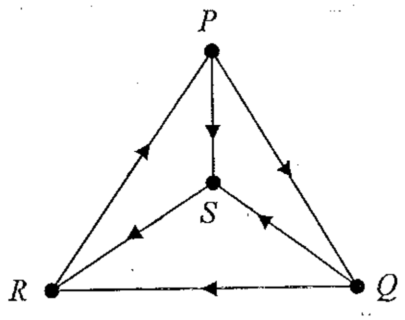
08. Let $D = (V, A)$ be a directed graph of order n and size m. Let $V = \{v_1, v_2, \ldots, v_n\}$ be the set of vertices of D. State the *Handshaking dilemma* for the directed graph D.

(a) Draw a simple digraph with *indegrees* 0, 1, 2 and *outdegree* 2, 1, 0.

(b) Are these two digraphs D_1 and D_2 *isomorphic*? Justify your answer.

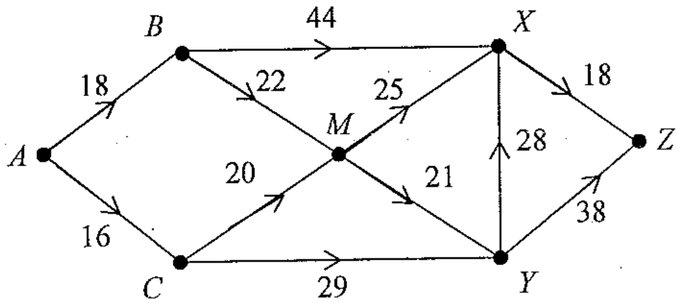


D_1



D_2

(c) Let the following digraph D represents a construction of a complete house, where A and Z represent the beginning and the completion of the job. Assume that the entire job cannot be completed until each path from A to Z has been traversed.



D

(i) Obtain the *critical path* from A to Z,

(ii) Verify the *Handshaking dilemma*.