The Open University of Sri Lanka
B.Sc/B.Ed. DEGREE, CONTINUING EDUCATION PROGRAMME
Final Examination - 2016/2017
Level 05 - Applied Mathematics
APU3147/APE5147– Statistical Inference

Duration: - Two Hours.

DATE: - 17-01-2018 **Time: - 1.30 p.m. to 3.30 p.m.**

Non programmable calculators are permitted. Statistical tables are provided.

Answer only four questions.

1.

(a) Briefly explain the following terms.

(i) Point estimation

(ii) Interval estimation

(b) Let $X_1, X_2, X_3, \ldots X_n$ be a random sample from a distribution with density given by $f(x;\theta)$. Let $\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \hat{\theta}_4,$ are functions of $X_1, X_2, X_3, \ldots X_n$. Suppose $\hat{\theta}_1, \hat{\theta}_2$ are unbiased estimators for parameter θ, $\hat{\theta}_3$ consistent estimator for parameter θ, $\hat{\theta}_4$, is the likelihood estimator for parameter θ and $MSE(\hat{\theta}_3) < MSE(\hat{\theta}_2)$. State whether the following statements are true or false. Justify your answer.

(i) $Bias(\frac{\widehat{\theta_1}+\widehat{\theta_2}}{2}) = 0$

(ii) $Var(\hat{\theta}_2) < Var(\hat{\theta}_3)$

(iii) Let $L(\theta)$be the likelihood function of θ. $L(\theta = \hat{\theta}_4) < L(\theta = \hat{\theta}_1)$.

(iv) $\hat{\theta}_2$ is an accurate estimator only for large samples.

(v) $\hat{\theta}_3$ is an accurate and precise estimator for large samples.

2.

Suppose weight of a certain product X, produced by ABC Company, follows normal distribution. However, the mean weight and variance weight of randomly selected product is unknown.
Let $X_1, X_2, X_3, \dots X_n$ be randomly selected products in grams.

(i) Find the Maximum likelihood estimators for the mean and the variance of a randomly selected product.

(ii) Weights of 16 randomly selected products in grams are given bellow. Using part (i) estimate the mean and the variance weight of a randomly selected product.

100.3	97.3	99.1	100.4	98.1
98.5	101.0	99.0	99.9	100.4
101.2	98.9	100.4	99.5	100.0
100.00				

(iii) Using a suitable statistical test comment on the claim that "variance of a randomly selected product is 2.5 g^2". Use 5% level of significance.

3.

In a particular school Mathematics mark of a randomly selected student in grade ten is denoted by X. Suppose $X \sim N(\mu, \sigma^2)$, μ and σ^2 are unknown. Mathematics marks of randomly selected 20 students in grade ten are collected. Sample mean of the Mathematics marks ($\bar{x}$) and standard deviation of the Mathematics marks (s) are given bellow.

Variable	n	$\bar{x}$	s
Marks	20	48.18	11.99

(a) Find 95% confidence interval for mean Mathematics mark of a randomly selected student in grade ten and interpret the results.

(b) Find 90% confidence interval for variance Mathematics mark of a randomly selected student in grade ten and interpret the results.

(c) Using a suitable statistical test comment on the claim that "mean Mathematics mark of a randomly selected student in grade ten is 50".

4.

In a production process of automotive crankshaft bearings, the production manager is interested in the proportion θ of nonconforming automotive crankshaft bearings produced. Suppose total production on a particular day is 10000. Random sample of 80 automotive crankshaft bearings (drawn with replacement) were tested on this particular day by the production manager. Suppose that 8 of the bearings had surface finish that is rougher than the specifications will allow.

(i) Construct a 95% confidence interval for θ

(ii) Construct 95% confidence interval for the total number of nonconforming automotive crankshaft bearings in the production on that day. Hence comment on the claim that " total number of nonconforming automotive crankshaft bearings produced on that particular day is 110"

(iii) Using a suitable statistical test comment on the claim that "proportion θ of nonconforming automotive crankshaft bearings produced on that particular day is greater than 0.11"

(iv) Using Part (iii) test the validity of the claim that " total number of nonconforming automotive crankshaft bearings produced on that particular day exceeds 110"

5.

(a) Briefly explain the following terms.

(i) Significance level.

(ii) Power of a statistical test.

(b)

Let $X_1, X_2, X_3, \ldots X_n$ be a random sample from a uniform distribution with density given by

$$f(x;\theta) = \frac{1}{\theta} \quad ; \; 0 \leq x \leq \theta ; \quad \theta > 0$$

(i) Prove that he mean of the above distribution is $\frac{\theta}{2}$.

(ii) Derive a moment estimator for θ. Is the moment estimator derived by you, an unbiased estimator for θ? Prove your answer.

(iii) Derive the maximum likelihood estimator for θ and for the mean of the above distribution.

(iv) A random sample drawn from the above distribution is given below.

0.49	1.38	1.95	1.28	1.80
0.21	0.05	0.48	0.49	1.40
0.59	0.23	0.26	1.65	1.43
1.56	0.16	0.82	0.37	0.80

[i] Estimate θ using moment estimator derived in part (ii).

[ii] Estimate θ and mean of the above distribution using maximum likelihood estimators derived in part (iii)

6.

(a) Briefly explain the following terms.

(i) Accuracy

(ii) Precision

(iii) Sampling distribution.

(a) Assignment marks and final examination marks of a particular subject for 15 students are given below.

Student Name	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Assignment mark	60	47	60	56	47	27	45	61	68	62	35	53	57	25	31
Final Mark	60	55	56	63	40	0	41	50	86	77	41	51	44	32	35

Using suitable statistical test, test the validity of the claim that "Expected assignment mark is greater than the expected final examination mark for a randomly selected student". Use 5% level of significance.