THE OPEN UNIVERSITY OF SRI LANKA
DEPARTMENT OF PHYSICS
BACHELOR OF SCIENCE DEGREE PROGRAMME -2019/2020 - LEVEL 05

PHU5312/PYU3173 – SOLID STATE PHYSICS Final Examination – 2019/2020

TIME: TWO HOURS (2 hrs) ANSWER FOUR QUESTIONS ONLY

Date: 06-01-2020 Time: 1.30 pm – 3.30 pm

1) (a) State what is meant by the following terms:

(i) Lattice vectors

(ii) Unit cell

(iii) Miller indices **(9 Marks)**

(b) A certain lattice is described by the following vectors:

$$\vec{a} = 2a\,\hat{i}, \quad \vec{b} = a(\hat{i} + \hat{j}), \quad \vec{c} = 2a\,\hat{k}$$

Where $\hat{i}, \hat{j}, \hat{k}$ are Cartesian unit vectors and a is a constant.

(i)Find the volume of the unit cell.

(ii)Find the reciprocal lattice vectors.

(iii)Find the volume of the unit cell in the reciprocal lattice.

(iv)Determine the angle between the (111) and (002) planes. **(16 Marks)**

2) (a) Write down the Laue equations for diffraction of X-rays by a crystalline solid. **(05 marks)**

(b) Show that the Bragg's equation is special case of the Laue equations. **(10 marks)**

(c) An X-ray analysis of a crystal is made with monochromatic X-rays of wavelength 0.58 Å. Bragg's reflections are obtained at angles (θ) of 6.45^0, 9.15^0 and 13^0 respectively. Determine the interplanar spacing of the crystal. **(10 marks)**

3) Assume the energy of two particles in the field of each other is given by the function

$$U(r) = -\frac{a}{r} + \frac{b}{r^8}$$

where a and b are constants, and r is the distance between the centres of the particles.

(a) The two particles form a suitable compound at the equilibrium separation, i.e. at $r = r_o$,

show that $$r = r_o = \left[\frac{8b}{a}\right]^{\frac{1}{7}}$$

(b) Show that the potential energy of the two particles in the stable configuration is equal to

$$-\frac{7}{8}\left(\frac{a}{r_0}\right)$$

(c) In a stable configuration, show that the energy of attraction is 8 times the energy of repulsion.

(d) Show that if the particles are pulled apart, the molecule will break as soon as,

$$r = \left[\frac{9}{2}\right]^{\frac{1}{7}} r_0$$

(e) Hence, show that the minimum force required to break the molecule is equal to

$$\frac{7}{9}\left[\frac{a^{\frac{9}{7}}}{(36b)^{\frac{2}{7}}}\right]$$ **(25 Marks)**

4) A simple, one dimensional model of a solid consists of a series of two atoms having masses m and M respectively, joined by springs of spring constant β, with an equilibrium separation of a.

(a) Write down an equation of motion for the $2n^{\text{th}}$ atom in terms of the displacement of the $2n-1$, $2n$ and $2n+1$ atoms. **(4 Marks)**

(b) If the solutions to the equations of motion for a diatomic linear chain are given by

$$\omega^2 = \beta\left(\frac{1}{m} + \frac{1}{M}\right) \pm \beta\sqrt{\left(\frac{1}{m} + \frac{1}{M}\right)^2 - \frac{4\sin^2(ka)}{mM}}$$

Sketch the dispersion relation $\omega(k)$ for a diatomic linear chain, indicting the Optical branch, Forbidden gap and the Acoustic branch clearly **(10 Marks)**

(c) Show as $q \to 0$, the ω_+ corresponds to Optical branch is given by the relation

$$\omega_+ = \sqrt{2\beta\left(\frac{1}{m} + \frac{1}{M}\right)}$$ and

when $q = \frac{\pi}{2a}$, ω_+ corresponds to Optical branch is given by the relation

$$\omega_+ = \sqrt{\frac{2\beta}{m}}$$ **(6 Marks)**

(d) Assuming that for NaCl, m = 3.82 x10^{-26} kg and M = 5.89 x10^{-26} kg, and the minimum energy separation between the optical and acoustic branches is 7.26×10^{-22} J determine the spring constant β of the spring. **(5 Marks)**

5) (a) In the presence of an applied electric field E across a specimen of a metal, free electrons of the metal tend to drift slowly in a direction opposite to that of the applied electric field. If τ is the average time between any two collisions, show that

$$J = \frac{ne^2\tau}{m}E$$ **(10 Marks)**

Where J is the current density, E is the applied electric field, n is the number of electrons per unit volume, and e and m are the electron charge and mass respectively.

(b) A steady current of 50 A is flowing through a copper wire of cross sectional area 10^{-6} m^2. Assuming each copper atom gives off one free electron calculate.

- (i) the density of free electrons
- (ii) the effective speed of a free electron with its kinetic energy equal to the Fermi energy of copper, which is 7.0eV.
- (iii) the average drift speed of electron.
- (iv) the average time between collisions for free electrons in copper.
- (v) the mean free path of free electrons

The density of copper is 8.92×10^3 kg m^{-3} and its atomic weight is 63.5. The resistivity of copper is $1.7 \times 10^{-8}\ \Omega$ m. **(15 Marks)**

6) (a) Sketch the form of the Fermi-Dirac distribution function $f(E)$ for the electron gas at

(i) $T = 0$ K and (ii) $T > 0$ K.

Indicate on your plot the location of the Fermi energy E_F. **(4 Marks)**

(b) Assuming that the density of states is given by

$$g(E)dE = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{1/2}dE$$

- (i) In the usual notation, show that the Fermi energy is given by

$$E_F = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3}$$

where n is the number of electrons per unit volume.

- (ii) Show also that the internal energy per unit volume of the electron gas is

$$U = \frac{3}{5}nE_F$$ **(12 Marks)**

(c) Potassium has density 828 kg m^{-3} and unit valency, determine

- (i) the number of conduction electrons per m^3 in potassium.
- (ii) the Fermi energy of the electron gas.
- (iii) the internal energy of the electron gas in 1kg of potassium.

(9 Marks)

Assume according to Fermi-Dirac statistics, the probability that an electron has an energy E at a temperature T is given by the distribution function

$$f(E) = \frac{1}{e^{\frac{E-E_F}{k_B T}} + 1}$$

where k_B is the Boltzmann constant and E_F is the Fermi energy.

Physical Constants

electron charge	$e = 1.60\times10^{-19}$ C
electron mass	$m_e = 9.11\times10^{-31}$ kg $= 0.511$ MeV c^{-2}
proton mass	$m_p = 1.673\times10^{-27}$ kg $= 938.3$ MeV c^{-2}
neutron mass	$m_n = 1.675\times10^{-27}$ kg $= 939.6$ MeV c^{-2}
Planck's constant	$h = 6.63\times10^{-34}$ J s
Dirac's constant ($\hbar = h/2\pi$)	$\hbar = 1.05\times10^{-34}$ J s
Boltzmann's constant	$k_B = 1.38\times10^{-23}$ J K^{-1} $= 8.62\times10^{-5}$ eV K^{-1}
speed of light in free space	$c = 299\,792\,458$ m s^{-1} $\approx 3.00\times10^{8}$ m s^{-1}
permittivity of free space	$\varepsilon_0 = 8.85\times10^{-12}$ F m^{-1}
permeability of free space	$\mu_0 = 4\pi\times10^{-7}$ H m^{-1}
Avogadro's constant	$N_A = 6.02\times10^{23}$ mol^{-1}
gas constant	$R = 8.32$ J mol^{-1} K^{-1}
ideal gas volume (STP)	$V_0 = 22.4$ l mol^{-1}
gravitational constant	$G = 6.67\times10^{-11}$ N m^2 kg^{-2}
Rydberg constant	$R_\infty = 1.10\times10^{7}$ m^{-1}
Rydberg energy of hydrogen	$R_H = 13.6$ eV
Bohr radius	$a_0 = 0.529\times10^{-10}$ m
Bohr magneton	$\mu_B = 9.27\times10^{-24}$ J T^{-1}
fine structure constant	$\alpha \approx 1/137$
Wien displacement law constant	$b = 2.898\times10^{-3}$ m K
Stefan's constant	$\sigma = 5.67\times10^{-8}$ W m^{-2} K^{-4}
radiation density constant	$a = 7.55\times10^{-16}$ J m^{-3} K^{-4}
mass of the Sun	$M_\odot = 1.99\times10^{30}$ kg
radius of the Sun	$R_\odot = 6.96\times10^{8}$ m
luminosity of the Sun	$L_\odot = 3.85\times10^{26}$ W
mass of the Earth	$M_\oplus = 6.0\times10^{24}$ kg
radius of the Earth	$R_\oplus = 6.4\times10^{6}$ m

Conversion Factors

1 u (atomic mass unit) $= 1.66\times10^{-27}$ kg $= 931.5$ MeV c^{-2}	1 Å (angstrom) $= 10^{-10}$ m
1 astronomical unit $= 1.50\times10^{11}$ m	1 g (gravity) $= 9.81$ m s^{-2}
1 eV $= 1.60\times10^{-19}$ J	1 parsec $= 3.08\times10^{16}$ m
1 atmosphere $= 1.01\times10^{5}$ Pa	1 year $= 3.16\times10^{7}$ s

****