00148

THE OPEN UNIVERSITY OF SRI LANKA
BACHELOR OF TECHNOLOGY HONORS IN ENGINEERING /
BACHELOR OF SOFTWARE ENGINEERING HONORS – LEVEL 05
FINAL EXAMINATION – 2015/2016
MPZ5140/ MPZ5160 – DISCRETE MATHEMATICS II
DURATION: THREE (03) HOURS.

Date: 16th November 2016 **TIME: 1330hrs – 1630 hrs.**

Instruction:

- **Answer only six questions.**
- **Please answer a total of six questions choosing at least one from each single section.**
- **All symbols are in standard notation and state any assumption that you made.**

SECTION – A

01. i. Let $(S,*)$ be an algebraic system such that $(a * b) * a = a$ for all $a, b \in S$. Show that $a * (a * b) = a * b$ for all $a, b \in S$. [10%]

ii. Define a semi-group in usual notation.

Let " $*$ " be a binary operation on $\mathbb{Z}$ defined by the following two ways:

a) $x * y = x - y$ for all $x, y \in \mathbb{Z}$.

b) $a * b = a(1 + b) + b(1 + a)$ for all $a, b \in \mathbb{Z}$.

Verify that whether $(\mathbb{Z},*)$ is a semi- group for each of the above case. [50%]

iii. Let " $*$ " be a binary operation defined on $S = \mathbb{Q} \times \mathbb{Q}$, the set of all ordered pairs of rational numbers, by $(a, b) * (x, y) = (ax, ay + b)$ for all $(a, b), (x, y) \in S$. Show that (S,$*$) is a semi-group or not. [40%]

02. i. Define a group and an Abelian group with usual notation. [15%]

ii. Prove that $\mathbb{Z}$ is an abelian group with respect to the binary operation " $*$ ". Defined by $x * y = x + y + 5xy$, for all $x, y \in \mathbb{Z}$. [60%]

iii Consider the group (G_1,$*$), where $G_1 = \left\{ \begin{pmatrix} a & -b \\ b & a \end{pmatrix} \mid a^2 + b^2 \neq 0,\ a, b \in \mathbb{R} \right\}$. and " $*$ " denotes the usual matrix multiplication. Show that(G_1,$*$) is an Abelian group. [25%]

03. i. Define a Homomorphism and Isomorphism for groups in usual notation. [30%]

ii. Let $(\mathbb{Z}, +)$ and $(\mathbb{Z}', +)$ be the groups of integers and even integers. Define a map $f: \mathbb{Z} \rightarrow \mathbb{Z}'$ such that $f(x) = 2x$ all $x \in \mathbb{Z}$.

Show that f is an Isomorphism.

[40%]

iii. Show that the mapping $\emptyset$ from the symmetric group p_n onto the multiplicative group $G_2 = \{-1,1\}$, defined by $\emptyset(\alpha) = 1$ or -1, according as α is even or odd permutation in p_n, is a homomorphism of p_n onto G_2. [30%]

SECTION – B

04. i. By drawing each of the following graphs, determine which of the following graphs are simple.

a) $G_2 = \{V_1, E_1\}$ where $V_1 = \{1,2,3, \dots, 9,10\}$ and

$E_1 = \{\{x, y\} | \, x + 2y$ is odd and $x < y\}$ [15%]

b) $G_2 = \{V_2,\ E_2\}$, where $V_2 = \{1,2,3,4,5,6,7,8,9,10\}$ and

$E_2 = \{\{i, j\} | i \times j$ is a divided by 5 and $i \neq j\}$ [15%]

c) $G_3 = \{V_3, E_3\}$, where $V_3 = \{v, v_2, v_3, v_4, v_5, v_6, v_7\}$ and

$E_3 = \{\rho_1, \rho_2, \rho_3, \rho_4, \rho_5, \rho_6\}$ with $\rho_1 = (v_1, v_4)$, $\rho_2 = (v_1, v_2)$,

$\rho_3 = (v_2, v_3)$, $\rho_4 = (v_4, v_3)$, $\rho_5 = (v_5, v_7)$, and $\rho_6 = (v_2, v_5)$. [15%]

ii. Draw all possible simple, connected graphs with 6 vertices and 15 edges. [10%]

iii Find the number of vertices of a complete graph which has at least 1000 edges. [25%]

iv Show that there is no simple graph with 12 vertices and 28 edges in which

a) the degree of each vertex is either 3 or 4

b) the degree of each vertex is either 3 or 6. [20%]

05. i. Determine the number of edges in a graph with 6 vertices, 2 of degree 4 and 4 of degree 2. Draw two such graphs. [30%]

ii. For each of the following degree sequences given bellow, find a graph, if it exists. In the case of exist, draw a corresponding graph. Otherwise, give reasons for the non-existence.

a) 4, 4, 4, 3, 2 b) 3, 3, 3, 3, 2 c) 5, 4, 3, 2, 1, 1. [30%]

iii. Prove that a complete graph with n vertices contains $n(n-1)/2$ edges. [40%]

06. i. Draw the trees for the following cases:

a) twelve vertices, at least half of which has degree one, [15%]

b) six vertices exactly three of which has degree one. [15%]

ii. Let G be a graph whose adjacency matrix A is given by

$$A = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix}.$$

a) Let $V(G) = \{a_1, a_2, a_3, a_4\}$. Find the number of paths of length four (4) joining vertices a_3 and a_1. Give the list of paths if they exist. [40%]

b) Without drawing a diagram for G, determine whether G is connected or not. [20%]

c) Draw the graph corresponding to the adjacency matrix A. [10%]

SECTION – C

07. i. For the given relationship $x_{n+1} = \lambda x_n(1 - x_n)$, where $\lambda = 1.6$, obtain the convergent value in the for 4 digits each of the following case:

a) $x_0 = 0.3$ b) $x_0 = 0.6$ [40%]

ii. Iterate the Eco-system growth model relationship $t_{n+1} = \lambda t_n(1 - t_n)$, where $\lambda = 2.1$ and $t_0 = 0.3$ (at least 6 iteration steps are necessary) and draw the graph for the relation. [20%]

iii Iterate the relation $Z_{n+1} = Z_n^2 + \lambda$, where $\lambda = 0$ (at least 5 iteration steps are necessary), and draw the diagram for each of the following case:

a) $Z_0 = 2.2 + i0.5$ b) $Z_0 = 0.3 + i0.5$

Hence, find Z_n as $n \to \infty$ for each case. [40%]

08. A three dimensional system is governed by the following system of differential equations:

$$\frac{dy_1}{dt} = y_1 + 2y_2 - y_3$$

$$\frac{dy_2}{dt} = y_1 + y_3$$

$$\frac{dy_3}{dt} = 4y_1 - 4y_2 + 5y_3,$$

where $y_1, y_2,$ and y_3 are function of t and at $t = 0$, $(y_1, y_2, y_3) = (1, 1, 0)$.

Find the phase space value $\{y_1(t), y_2(t), y_3(t)\}$ for $t = 1, 2$. [100%]

09. i. Consider the grammar G with $V = \{S, A, B\},\ \Sigma = \{a, b\}$ and $P = \{S \rightarrow aABa,\ A \rightarrow baABb,\ B \rightarrow Aab,\ aA \rightarrow baa\}$ in the usual notation. Verify whether the string $w = baabbabaaababa \in L(G)$. [25%]

ii. Consider the language $L_1 = \{1, 2, 12\}$ and $L_2 = \{a, b, ab\}$. Find L_1L_2, and L_2^3.

iii Let M be a Mealy machine with the following functions. Let $s \in S, a \in I$ and $x \in I^*$. [30%]

$\delta: S \times I^* \rightarrow S$ and $\beta^*: S \times I^* \rightarrow O^*$ by
$\delta^*(s, \Omega) = s,$
$\delta^*(s, a.x') = \delta^*[\delta(s, a), x'],$
$\beta^*(s, \Omega) = \Omega,$
$\beta^*(s, a.x') = \beta(s.a)\beta^*[\delta(s, a), x'].$

The two frame binary pipeline devise hold up two binary as in the following table.

State	Input		Output	
	0	1	0	1
000	010	001	0	1
001	101	001	1	0
010	011	100	0	0
011	010	011	0	1
100	100	111	1	1
101	101	110	1	0
110	110	101	1	1
111	111	101	0	1

Find the two frame binary pipeline buffer and work out its response to the sequence 011011 from the state 100. [45%]

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