

The Open University of Sri Lanka

B.Sc/B.Ed. DEGREE, CONTINUING EDUCATION PROGRAMME

~~Open~~ ^{No} Book Test (~~Open~~ ^{NOT} ~~Book~~ Test) (OBT) 2023/2024

Level 05 - Applied Mathematics

ADU5301– Regression Analysis I

Date: 10.02.2024

10.30 a.m. to 11.30 a.m.

Instructions

- This examination is of **one-hour** duration.
- Answer **all** questions.
- Each of the two questions is allocated equal marks, distributed as indicated. Final total mark will be converted to a mark out of 100.
- Non programmable calculators are permitted.
- Where appropriate, you may use $\hat{\beta}_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} = \frac{\sum (x_i y_i) - (\sum x_i)(\sum y_i)/n}{\sum x_i^2 - (\sum x_i)^2/n}$ and $\hat{\beta}_0 = \frac{\sum y_i}{n} - \hat{\beta}_1 \frac{\sum x_i}{n}$, in the usual notation.

1. a) Reaction times (in minutes) of a chemical reaction y , were measured on 40 samples, by adding known amounts of a catalyst (mg), x . Amounts of catalyst added to samples were 0 (no catalyst), 0.2, 0.4, 0.6 and 0.8 milligrams. Eight replicates were collected with each amount of catalyst. Summary statistics calculated from the data are presented below.

$$\sum x_i = 16, \quad \sum y_i = 125.0, \quad \sum x_i^2 = 9.6 \quad \sum y_i^2 = 414.78, \quad \sum x_i y_i = 41.3.$$

Researcher wants to fit a simple linear regression model, using amount of catalyst as the predictor variable.

- Calculate the least squares estimates for the slope and intercept parameters in the model equation and obtain the equation of the fitted line.
(40 marks)
- Based on the fitted model, estimate the change in the mean reaction time, when amount of catalyst is increased from 0.3 mg to 0.5 mg.
(20 marks)
- Based on the fitted model, a student concluded that, when the amount of catalyst is increased from 0.8 mg to 1.2 mg, change in the mean reaction time would be approximately 11 minutes. Comment on the conclusion made by the student.
(20 marks)

iv) A new sample that had received 0.4 mg of catalyst, had a reaction time of 3 minutes. Researcher wants to add the new observation to the previous data and refit the regression model, using all the data values. Since the researcher had collected the new observation at the average of the previously used x values, a student stated that the fitted line obtained in part (i), will not change due to the addition of the new observation. Do you agree with the statement made by the student? If you agree, explain why the fitted line will not change. If you disagree, clearly explain how the fitted regression line would be changed..

(20 marks)

2. A researcher fitted the model $y = \beta_0 + \beta_1 x^2 + \epsilon$, for the monthly increment in height (cm), y , of a certain medicinal plant, where x denote the age of the plant (in weeks). Least squares estimates for the slope and intercept parameters for the model fit were 31.3 and -0.01 respectively. Let \hat{y} and $\hat{\epsilon}$ denote the fitted values and residuals respectively.

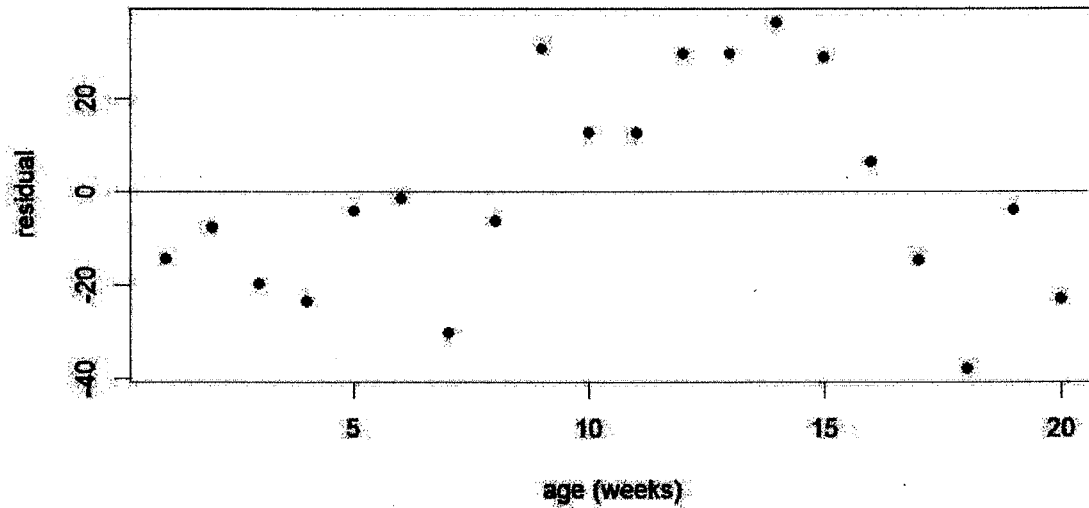
i) Estimate \hat{y} and $\hat{\epsilon}$ of a 10-week old plant. Clearly explain what these values represent, in relation to this study.

(30 marks)

ii) Since the estimate for the slope parameter is negative, a student stated that the monthly increment in height decreases with age. Comment on the statement made by the student.

(20 marks)

iii) . A plot of least squares residuals obtained from the above model fit, against age of the plant is given below.



State whether each of the following conclusions drawn by a student, based on the above plot, is true or false.

- Observations collected on plants that are less than 5 weeks of age, have small random errors compared to observations collected at other ages.
- Random errors are normally distributed.
- Plot indicates that the fitted regression function is not appropriate.
- Since researcher has fitted a model that is not linear, least squares residuals cannot sum to zero.
- Fitted model overestimates the height increment in 20-weeks old plants..

(5x10 marks)

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