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The Open University of Sri Lanka
B.Sc. /B.Ed. Degree Programme
Applied Mathematics - Level 05
ADU5304 - Operational Research
No Book Test (NBT) - 2023/2024

Duration: One Hour

Date: 03.02.2024 **Time: 01.00 p.m. - 02.00 p.m.**

ANSWER ALL QUESTIONS.

Question 1

The mean rate of arrival of planes at an airport during the peak period is 20 per hour, but the actual number of arrivals is any hour follows a poisson distribution. The airport can land 60 planes per hour on an average in good weather, or 30 per hour in bad weather, but the actual number landed in any hour follows a poisson distribution with a respective average. When there is congestion, the planes are forced to fly over the field in the stack awaiting the landing of other planes that arrived earlier.

a) How many planes would be flying over the stack on an average in good weather and in bad weather?
b) How long a plane would be in the stack and the process of landing in good and bad weather?
c) How much stack landing time to allow so that priority to land out of order would have to be requested only one time in twenty.

Question 2

A one person barber shop has three chairs for customers. Assume that the customers arrive in poisson distribution at a rate of 4 per hour and the barber services customers according to an exponential distribution with mean of 10 minutes. Further, if a customer arrives and there are no empty chairs in the shop, he will leave.

a) What is the probability that a customer can get directly into the barber chair upon arrival?
b) What is the expected number of customers waiting for a haircut?

Formulas (in the usual notation)

(M/ M / 1): (∞ / FIFO)

$$P_n = \left(\frac{\lambda}{\mu}\right)^n \left(1-\frac{\lambda}{\mu}\right)$$

$$E(n) = \frac{\lambda}{\mu-\lambda}, \qquad E(m) = \frac{\lambda^2}{\mu(\mu-\lambda)}$$

$$E(w) = \frac{\lambda}{\mu(\mu-\lambda)} \quad \text{and} \quad E(v) = \frac{1}{\mu-\lambda}.$$

(M/M/I): (N/FIFO) Oueueing

$$P_0 = \begin{cases} \dfrac{(1-\rho)\rho^n}{1-\rho^{N+1}}, & \rho \neq 1 \\ \dfrac{1}{N+1}, & \rho = 1 \end{cases}$$

$$E(m) = \frac{\rho^2[1-N\rho^{N-1}+(N-1)\rho^N]}{(1-\rho)(1-\rho^{N+1})}$$

$$E(w) = E(v) - \frac{1}{\mu}$$

$$E(n) = \frac{\rho[1-(N+1)\rho^N + N\rho^{N+1}]}{(1-\rho)(1-\rho^{N+1})}$$

(M/M/C) : (∞/FIFO) Model

$$P_0 = \left[\sum_{n=0}^{C-1}\frac{1}{n!}\rho^n + \rho C\sum_{n=C}^{\infty}\frac{1}{C!}\left(\frac{\rho}{C}\right)^{n-C}\right]^{-1}$$

$$= \left[\sum_{n=0}^{C-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{C!}\left(\frac{\lambda}{\mu}\right)^C \frac{C\mu}{C\mu-\lambda}\right]^{-1}$$

$$E(w) = \frac{1}{\lambda}E(m) = \frac{\mu\left(\frac{\lambda}{\mu}\right)^C P_0}{(C-1)!(C\mu-\lambda)^2}.$$

$$E(v) = E(w) + \frac{1}{\mu} = \frac{\mu\left(\frac{\lambda}{\mu}\right)^C P_0}{(C-1)!(C\mu-\lambda)^2} + \frac{1}{\mu}.$$