

The Open University of Sri Lanka

B.Sc/B.Ed. DEGREE, CONTINUING EDUCATION PROGRAMME

Open Book Test - 2023/2024

Level 05 - Applied Mathematics

ADU5305/ADE5305– Statistical Inference

Duration: - One hour.



DATE: - 23-12-2023

Time: - 9.00 a.m. – 10.00 a.m.

Non programmable calculators are permitted.

Answer all questions

1.

Let $X_1, X_2, X_3, \dots, X_n$ be a random sample from a distribution with density given by $f(x; \theta)$.

Let $\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \hat{\theta}_4$ are functions of $X_1, X_2, X_3, \dots, X_n$. Suppose $\hat{\theta}_1, \hat{\theta}_2$ unbiased estimators for parameter θ , $\hat{\theta}_3$ consistent estimator for parameter θ . State whether the following statements are true or false. Justify your answer.

- (i) $\text{Bias}\left(\frac{\hat{\theta}_1}{2} + \frac{\hat{\theta}_2}{2}\right) > 0$
- (ii) $\text{Var}(\hat{\theta}_3) = 0$ for large samples
- (iii) $\frac{\hat{\theta}_1 + \hat{\theta}_4}{2}$ is an accurate estimator for large samples
- (iv) $\hat{\theta}_2$ is a consistent estimator for θ .
- (v) $\hat{\theta}_3$ is an accurate and precise estimator for large samples.

2.

Let $X_1, X_2, X_3, \dots, X_n$ be a random sample from a uniform distribution with density given by

$$f(x; \theta) = \frac{1}{2 - \theta} \quad ; \quad 1 \leq x \leq 3 - \theta; \quad 0 < \theta < 1.$$

- (i) Find the mean of the above distribution.
- (ii) Derive Moment estimator for mean of the above distribution.
- (iii) Derive Maximum likelihood estimator for mean of the above distribution.
- (iv) A random sample drawn from the above distribution is given in the following table.

2.39	1.90	2.16	1.67	2.15
1.05	1.92	1.82	2.15	1.92
2.43	1.59	1.58	2.07	1.38
1.19	1.35	2.38	1.90	1.74
1.33	1.98	2.14	1.88	1.12

- i. Estimate the mean of the above distribution using moment estimator derived in part (i).
- ii. Estimate the mean of the above distribution using maximum likelihood estimator derived in part(ii).