

The Open University of Sri Lanka  
 B.Sc/ B.Ed Degree Programme  
 Applied Mathematics – Level 05  
 ADU5308 – Graph Theory  
 CAT I – Open Book Test - 2023/2024



**DURATION: ONE HOUR**

**Date: 30.12.2023**

**Time: 1.00p.m. – 2.00p.m.**

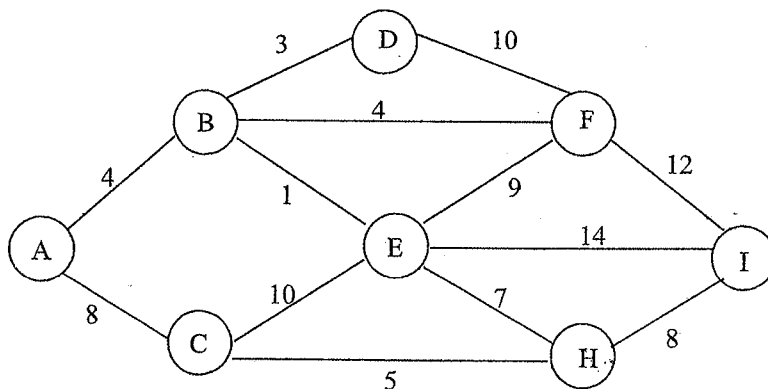
ANSWER ALL QUESTIONS.

1.

- Can a graph have 7 vertices of degrees 1, 1, 1, 2, 2, 3 and 6?  
 Either draw such a graph or explain why it cannot exist.
- Do all the complete graphs have Hamiltonian cycles? Justify your answer.
- Let  $A, B$  and  $C$  be three distinct vertices of a weighted graph  $G$  and let  $d(A, B)$  be the distance between  $A$  and  $B$ . Show that any solution to the travelling salesman problem for  $G$  has weight at least  $d(A, B) + d(B, C) + d(C, A)$ .

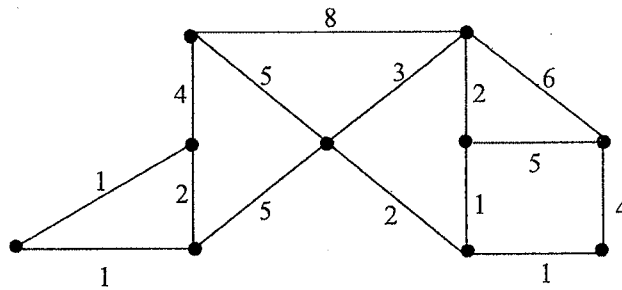
2.

- Use *Dijkstra's algorithm* to find the shortest path from  $A$  to  $I$  for the following graph  $G$ .



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- b) Display the labeling of the universal addresses system next to the vertices in the ordered rooted tree in the following lexicographic ordering of the labeling.  
 $0 > 1 > 1.1 > 1.1.1 > 1.2 > 1.2.1 > 1.2.2 > 2 > 3 > 3.1 > 3.1.1 > 3.1.1.1 > 3.1.2 > 3.2 > 3.3 > 4 > 4.1 > 4.2$
- c) Use *Kruskal's Greedy algorithm* to find the minimum weighted spanning tree of the following graph G.



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