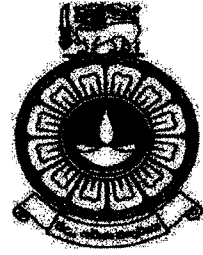


The Open University of Sri Lanka  
 B.Sc/B.Ed. Degree Programme – Level 05  
 Open Book Test (OBT) – 2023/2024  
 Pure Mathematics – Level 05  
 PEU5303 – Number Theory



**Duration: ONE HOUR**

Date: 05.01.2024

Time: 09.00 AM – 10.00 AM

**Answer All Questions**

1. (a). Let  $x$  and  $y$  be rational numbers. Prove that  $(x - y)$  is a rational number.
  - (b). Consider the following subsets of the set of real numbers. Which one has a least element? Justify your answer.
    - (i).  $(-2, 2]$
    - (ii).  $\{n: n \in \mathbb{N}, n \geq 2023\}$
  - (c). Using the Principle of Mathematical Induction, verify the identity  $3|n(2n^2 + 7)$ , for each  $n \in \mathbb{N}$ .
2. (a). Find  $x, y \in \mathbb{Z}$  such that  $\gcd(42823, 6409) = 42823x + 6409y$ .  
 Hence find  $\text{lcm}(42823, 6409)$ .  
**Note:** The greatest common divisor of  $a$  and  $b$  is denoted by  $\gcd(a, b)$  and the least common multiple of two nonzero integers  $a$  and  $b$  is denoted by  $\text{lcm}(a, b)$ .
    - (b). Consider the Diophantine equation  $3x + 5y = 1$ :
      - (i). Show that an integer solution exists for the equation.
      - (ii). Find a particular solution,  $x = x_0, y = y_0$ .
      - (iii). Write down the complete solution in terms of  $x_0$  and  $y_0$ .
    - (c). By using the theory of congruence, verify that  $43|6^{n+2} + 7^{2n+1}$ , for  $n \geq 1$ .