005

The Open University of Sri Lanka

B.Sc/B.Ed. Degree Programme – Level 05

Open Book Test (OBT) – 2023/2024

Pure Mathematics – Level 05

PEU5303 – Number Theory

Duration: ONE HOUR

Date: 05.01.2024 Time: 09.00 AM – 10.00 AM

Answer All Questions

1. (a). Let x and y be rational numbers. Prove that $(x - y)$ is a rational number.

   (b). Consider the following subsets of the set of real numbers. Which one has a least element? Justify your answer.

   (i). $(-2, 2]$

   (ii). $\{n: n \in \mathbb{N}, n \geq 2023\}$

   (c). Using the Principle of Mathematical Induction,

   verify the identity $3|n(2n^2 + 7)$, for each $n \in \mathbb{N}$.

2. (a). Find $x, y \in \mathbb{Z}$ such that $\gcd(42823, 6409) = 42823x + 6409y$.

   Hence find $\text{lcm}(42823, 6409)$.

   Note: The greatest common divisor of a and b is denoted by $\gcd(a, b)$ and the least common multiple of two nonzero integers a and b is denoted by $\text{lcm}(a, b)$.

   (b). Consider the Diophantine equation $3x + 5y = 1$:

   (i). Show that an integer solution exists for the equation.

   (ii). Find a particular solution, $x = x_0, y = y_0$.

   (iii). Write down the complete solution in terms of x_0 and y_0.

   (c). By using the theory of congruence, verify that $43|6^{n+2} + 7^{2n+1}$, for $n \geq 1$.