

The Open University of Sri Lanka
 B.Sc./B.Ed. Degree Programme
 Pure Mathematics - Level 04
 PEU4316 - Differentiable Functions
 Open Book Test (OBT) - 2023/2024



Duration: One Hour

Date: 05.01.2024

Time: 02.30 p.m. - 03.30 p.m.

ANSWER ALL QUESTIONS.

1) (a) Let $f(x) = \frac{2x+3}{x+1}$, $x \in \mathbb{R} \setminus \{-1\}$.

By using the definition of the derivative of a function at a point, find $f'(0)$.

(b) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by

$$f(x) = \begin{cases} \frac{x}{x+1} & x \geq 0 \\ x^2 & x < 0 \end{cases}$$

(i) Find the expressions for $\frac{f(x)-f(0)}{x-0}$ when $x < 0$ and $x > 0$.

(ii) Find $f'_-(0)$ and $f'_+(0)$.

(iii) Is the function $f(x)$ differentiable at point $x = 0$? Justify your answer.

2) Let $f(x) = \begin{cases} 2x - 9, & x \in (0, \infty) \cap \mathbb{Q}^c \\ \frac{x^2}{9}, & x \in (0, \infty) \cap \mathbb{Q} \end{cases}$ and $g(x) = \sqrt{x}$, $x \in [0, \infty)$ and

$h(x) = \frac{g(x)}{f(x)}$, $x \in (0, \infty)$. Show that h is differentiable at 9.