The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme
Pure Mathematics - Level 04
PEU4316 – Differentiable Functions
No Book Test (NBT) - 2023/2024

Duration: One Hour

Date: 09.02.2024   **Time: 04.00 p.m. - 05.00 p.m.**

ANSWER ALL QUESTIONS.

1) Let $a, b, c \in \mathbb{R}$ be such that $a^2 > 3b$. Let $f(x) = x^3 + ax^2 + bx + c$, $x \in \mathbb{R}$.

   Also let α, β be the roots of $3x^2 + 2ax + b = 0$ with $\alpha < \beta$.

   Show that (a) $f(x) - f(\beta) = (x - \beta)^2 \left(x - \alpha + \frac{\beta - \alpha}{2} \right)$

   (b) $f(x) - f(\alpha) = (x - \alpha)^2 \left(x - \beta + \frac{\alpha - \beta}{2} \right)$ for each $x \in \mathbb{R}$.

   Deduce that f has a local minimum at β and a local maximum at α.

2) State the Mean-Value Theorem for Derivatives.

   Using the above Theorem or otherwise show that for $a, b \in \mathbb{R}$ such that $0 < a < b$,

   $$\sqrt{a} - \sqrt{b} < \frac{b - a}{2\sqrt{a}}.$$

3) By applying L'Hospital Rule, compute the limit (if exists) of each of the following indeterminate forms.

   a. $\displaystyle \lim_{x \to 0} \frac{\sin x - x}{x^2}$

   b. $\displaystyle \lim_{x \to 1} \left(\frac{1}{x-1} - \frac{\ln x}{(x-1)^2} \right)$