

The Open University of Sri Lanka
 B.Sc/B.Ed. Degree Programme – Level 04
 Open Book Test– 2023/2024
 Pure Mathematics
 PEU4303 – Group Theory 1



Duration: - One Hour.

Date: -28 .12.2023

Time: - 01.00 p.m. – 02.00 p.m.

Answer All Questions.

(01) (a) Consider $r_{270} = \begin{pmatrix} A & B & C & D \\ D & A & B & C \end{pmatrix} \in D_8$. Find the order of r_{270} and hence find

$$r_{270}^6 \text{ and } r_{270}^{-1}.$$

(b) Find the order of $r_{180} = \begin{pmatrix} A & B & C & D \\ C & D & A & B \end{pmatrix}$ and hence find r_{180}^{-41} .

(c) Consider $r_{270}, r_{180} \in D_8$. Show that r_{270} and r_{180} commute and hence find

$$(r_{270} \circ r_{180})^3.$$

(d) Express the following symmetries in the form $s_y^i \circ r_{90}^j$.

$$(i) s_x \circ r_{90}^{-3} \quad (ii) r_{90}^{-5} \circ s_x.$$

(02) (a) Let $G = \mathbb{Z} \times \mathbb{Z}$ be a set and \circ be a binary operation on G defined by

$$(a, b) \circ (c, d) = (a + c, b + d).$$

Show that (G, \circ) is a group.

(b) Let $S = \{1, 2, 4, 5, 7, 8\}$ be a subset of \mathbb{Z}_9^\times .

Show that (S, \times_9) is a group and hence find the order of 4.