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The Open University of Sri Lanka

B.Sc/B.Ed. DEGREE, CONTINUING EDUCATION PROGRAMME

No Book Test: 2023/2024

Level 03 Pure Mathematics

PEU3202/PEE3202 Vector Spaces

Duration: - One Hour

Date: - 27-01-2024 **Time: 9.00 a.m. to 10.00 a.m.**

Answer all questions.

1.

(a) Let $M = \{(a,b,c,d) \mid a,b,c,d \in \mathbb{R}\}$. Note that M is a vector space over the field $\mathbb{R}$ under the usual addition and scalar multiplication.

Let the mapping $T : M \to M$ be defined by $T((a,b,c,d)) = ((a+b),\ b,\ 3c, d)$.

Determine whether the following sets are invariant subspaces of the vector space M over the field $\mathbb{R}$ under T.

(i) $W = \{(a,b,\ 0,0) \mid a,b \in \mathbb{R}\}$

(ii) $W = \{(a,0,0,c) \mid a,c \in \mathbb{R}\}$

(b)

Let $u = (x_1,x_2,x_3),\ v = (y_1,y_2,y_3)$ where $u,v \in \mathbb{R}^3$.

Define $< u,v > = x_1{}^2 - x_2{}^2 - x_1x_3$. Is $< u,v >$ an inner product on $\mathbb{R}^3$? Justify your answer.

2.

(a) Let u and v be any two vectors of a Euclidian Space.

(i) Prove that $\|u+v\| \leq \|u\| + \|v\|$

(ii) Define the angle between u and v

(iii) Suppose E^3 is the usual Euclidean three space and $u, v \in E^3$.

Let $u = (1,-1,2)$ and $v = (2,1,0)$. Find the angle between u and v

(b) Show that the three vectors $u_1 = (1,1,1)$, $u_2 = (0,1,1)$ and $u_3 = (0,0,1)$ form a basis for E^3, the usual Euclidean three space. Construct an orthogonal basis for E^3 out of $\{u_1, u_2, u_3\}$ using the Gram–Schmidt process.