The Open University of Sri Lanka

B.Sc/B.Ed. DEGREE, CONTINUING EDUCATION PROGRAMME

Open Book Test 2023/2024

Level 03 Pure Mathematics

PEU3202/PEE3202– Vector Spaces

Duration: - One hour

Date: -23-12-2023   **Time: 4.00 -5.00 p.m.**

Answer all questions

1.

(a) Let $V = \{ (a_1, a_2) \mid a_1, a_2 \in \mathbb{R} \}$. Is V a vector space over the field $\mathbb{R}$ under the operations given below? Justify your answer.

For any $(a_1, b_1), (a_2, b_2) \in V$ and $\alpha \in \mathbb{R}$

(i) $(a_1, b_1) + (a_2, b_2) = (1, a_2 + b_2)$; $\alpha (a_1, b_1) = (a_1, b_1)$

(ii) $(a_1, b_1) + (a_2, b_2) = (a_1 + b_1, a_2 + b_2)$; $\alpha (a_1, b_1) = (3a_1, \alpha^2 b_1)$

(b) Determine whether the set $V = \{ (2a, 3b) \mid a, b \in \mathbb{R} \}$ is a subspace of $\mathbb{R}^2$ over the field $\mathbb{R}$ under usual addition and scalar multiplication. Justify your answer.

(c) Is the three vectors $u_1 = (1, 2, 2)$, $u_2 = (1, -1, 2)$ and $u_3 = (1, 0, 1)$ form a basis for $\mathbb{R}^3$ over the field $\mathbb{R}$ under usual addition and scalar multiplication? Justify your answer.

2.

Let $V = \mathbb{R}^4$ and $W = \mathbb{R}^2$. Note that V and W are vector spaces over field $\mathbb{R}$ under the usual addition and scalar multiplication.

Consider the mapping $T : V \to W$ be defined by $T(a, b, c, d) = (a - b, c + 2d)$.

(i)   Show that T is a linear transformation.

(ii)   Find the kernel of T.

(iii)   Is T an Isomorphism? Justify your answer.

(iv)   Find the image of T and prove that image of T is a subspace of vector space W.