The Open University of Sri Lanka
B.Sc/B.Ed. DEGREE, CONTINUING EDUCATION PROGRAMME
Final Examination - 2023/2024
Level 04 Applied Mathematics
ADU4300/ADE4300– Statistical Distribution Theory
Duration: - Two Hours

Date: -21-10-2023 **Time: 9.30 a.m. to 11.30 a.m.**

Answer for four questions only. Statistical tables are provided

1. The random variable X denotes the lifetime (in years) of a certain electrical product produced by a company and it has the following density function.

$$f_X(x) = ke^{-kx};\ x > 0, k > o$$

The past data indicate that it is reasonable to take that the median of the lifetime to be five years.

(i) Calculate the value k.
(ii) Find the moment generating function of X.
(iii) Find the mean and the variance of the lifetime of the product.
(iv) Find the cumulative distribution function of the random variable lifetime of the product.
(v) What is the probability that a randomly selected product will fail within 7 years?

2.

(a) The traffic lights have three phases and there operating times were given as: ***stop 45%*** of the time, ***wait 10%*** of the time and ***go 45%*** of the time. Assuming that you only cross a traffic light when it is in the ***go*** position and that you have to pass 8 such traffic lights on your way to school.

(i) Model the number of times that you have to ***wait or stop*** on your way to school. State any assumptions that must be made and give possible values for the parameters.
(ii) Find the probability that you have to ***wait or stop*** at only three traffic lights on your way to school.
(iii) Find the probability that you have to ***wait or stop*** first time at the third traffic light on your way to school.
(iv) Find the probability that you have to ***wait or stop*** for the third time at the fifth traffic light on your way to school.

(b) In a study of the number of admissions to an emergency ward of a hospital on a Saturday morning during the period beginning at 12.00 midnight and ending at 2.00 a.m. is found to have an average of 3.5 admissions.
(i) What is the probability that none will be admitted during the above period?
(ii) What is the probability that two to six people (inclusive) will be admitted within the above period?

3.

Particular academic program has three levels and the average marks of ***level 1***, ***level 2*** and ***level 3*** of a student are given by the variables X_1, X_2 and X_3 respectively. The ***final average marks*** of a student is calculated by the formula,

$$final\ average \quad = \frac{X_1 + 2X_2 + 3X_3}{6}.$$

From the past experience it is known that $X_1 \sim N(50,100)$ $X_2 \sim N(45,100)$ and $X_3 \sim N(40,100)$. Assume that X_1, X_2 and X_3 are mutually independent:

(i) Calculate the expected ***final average marks*** of a student who enrolled in the above academic programme.
(ii) Calculate the variance of ***final average marks*** of a student who enrolled in the above academic programme.
(iii) Write the distribution of ***final average marks***. Clearly state the values of necessary parameters.
(iv) Find the proportion of students obtaining ***final average marks*** in between 40 and 60.
(v) Find the lowest ***final average marks*** of the highest 25% of the students.
(vi) Suppose that 750 students are enrolled to the above programme in 2023 batch. According to the rules and regulations of the above programme, student has to get a minimum of 40 for the ***final average marks***, to complete the programme successfully. Calculate the expected number of students who will complete the course in 2023 batch relying on past experience.

4.

(a) Suppose that X_1, X_2, X_3 are independent random variables described as
$X_1 \sim N(2,4)$ $X_2 \sim N(2,9)$ $X_3 \sim \chi_1^2$
Find the following probabilities. Show your calculations and state the justifications clearly..

(i) $\Pr[\left(\frac{X_2 - 2}{3}\right)^2 < 5.024\ ]$

(ii) $\Pr[2X_1 + 3X_2 + 4 > 18\ )$

(iii) $\Pr\left[\frac{\left(\frac{X_2 - 2}{3}\right)}{X_3} < 6.314\right]$

(b) The time between arrivals of customers at an automatic teller machine is an exponential random variable with the $exp(2)$ distribution. Find the probability that it will take less than three minutes to get the next three customers once a certain consumer arrives.

5.

A certain shop sells two brand of VCR, A and B. Let X denote the number of brand A, VCR machines sold per day and Y denote the number of brand B, VCR machines sold per day. The following table shows the joint probabilities, according to the past data.

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P(x,y)		x		
		0	1	2
y	0	0.10	0.04	0.02
	1	0.30	0.14	0.06
	2	0.08	0.2	k

(i) Find the value of k.
(ii) Find the marginal distribution function of Y.
(iii) What is the expected total number of sales of VCR on a randomly selected day?
(iv) What is the probability that the number of brand B VCR machines sold is more than that of brand A VCR machines on a randomly selected day,?
(v) On a particular day salesman of the shop has sold their first brand B VCR at 10.00 a.m. Assume that the shop opens at 9.00 a.m. and closes at 5.00 p.m. What is the probability of no sales of brand A VCR on that day?
(vi) Find the conditional probability mass function of brand A VCR machines sold on a randomly selected day given that at least one brand B VCR machine is sold on that day.
(vii) Are the sales of brand A and brand B independent? Justify your answer.

6)

(a) Describe in your own words the central limit theorem. Why is it such an important theorem in statistics?
(b) The diameter of certain components follows a normal distribution with mean and standard deviation 1.35 cm and 0.05 cm respectively. Suppose all components with diameters outside the range 1.25 to 1.45 are considered as defective components and are rejected.
(i) What proportion of components will be rejected in a batch of production? Give your answer rounded to the second decimal place.
(ii) Suppose sample of 10 components are selected.
I. Suggest a suitable probability distribution for sample mean. Clearly state the parameters of the distribution.
II. Find the probability that no detective items found in the sample.
(iii) Suppose that production items will be checked one by one.
I. Find the probability that first defective item found is the 5th checked item.
II. Find the probability that third defective item that found in the 9th inspection.

Table of Student t distribution

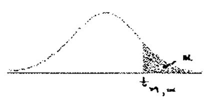

Table of $t_{\alpha,\nu}$ quantiles (t-table)

df ν	α 0.10	0.05	0.025	0.01	0.005	0.001	0.0005
1	3.078	6.314	12.706	31.821	63.657	318.309	636.619
2	1.886	2.92	4.303	6.965	9.925	22.327	31.599
3	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.61
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869

Table of Standard Normal Probabilities

Let $Z \sim N(0,1)$. This table contains the probabilities $Pr(Z \geq z)$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2297	0.2266	0.2236	0.2207	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1057	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084

Left tail values of Standard Gamma Table - W - gamma(α,1)

This table contain the probabilities Pr(W ≤ w)

	α					
w	1	2	3	4	5	6
1	0.393469	0.264241	0.080301	0.018988	0.00366	0.000594
2	0.632121	0.593994	0.323324	0.142877	0.052653	0.016564
3	0.77687	0.800852	0.57681	0.352768	0.184737	0.083918
4	0.864665	0.908422	0.761897	0.56653	0.371163	0.21487
5	0.917915	0.959572	0.875348	0.734974	0.559507	0.384039
6	0.950213	0.982649	0.938031	0.848796	0.714943	0.55432

Table of $\chi^2_{\alpha,\nu}$ quantiles (χ^2 table)

df	α							
ν	0.99	0.975	0.95	0.90	0.1	0.05	0.025	0.01
1	0	0.001	0.004	0.016	2.706	3.841	5.024	6.635
2	0.02	0.051	0.103	0.211	4.605	5.991	7.378	9.21
3	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345
4	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277
5	0.554	0.831	1.145	1.61	9.236	11.07	12.833	15.086

Let $X \sim \chi^2_\nu$ and α be a probability. This table contains the upper α quantiles $\chi^2_{\alpha,\nu}$ of the χ^2_ν distributions such that $\Pr(X > \chi^2_{\alpha,\nu}) = \alpha$.