The Open University of Sri Lanka

Department of Electrical and Computer Engineering

Final Examination 2015/2016

ECX5233 – Communication Theory & Systems

Time: 09.30 - 12.30 hrs. Date: 2016-12 -10

Answer Any Five Questions

1.

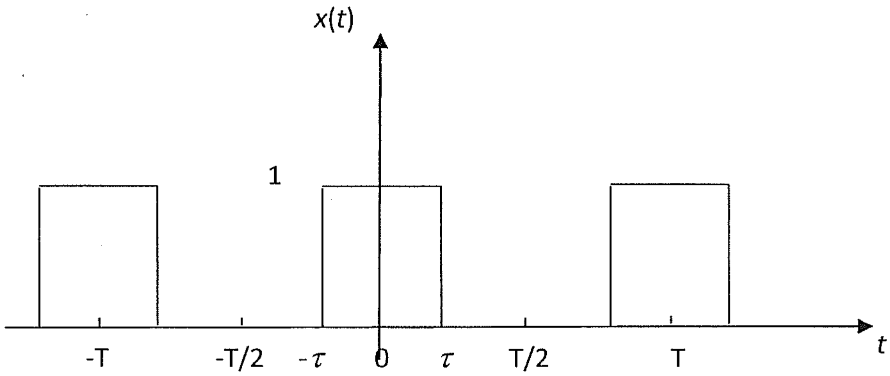


Fig.Q1

(a) Find the Fourier series of $x(t)$ (Express $x(t)$ in the form $x(t) = \sum A_n \cos n\omega_0 + B_n \sin n\omega_0$ and find A_n and B_n). Also show that A_n has a real value. What can you say about B_n? [5 *marks*]

(b) Plot $\sqrt{A_n^{\,2} + B_n^{\,2}}$ vs. $n\omega_0$, where $\omega_0 = \dfrac{2\pi}{T}$. [4 *marks*]

(c) How does the answer to (b) change if $x(t)$ is changed to $x(t - t_0)$? (No calculations expected for this answer.) [5 *marks*]

(d) If $T \to \infty$

(i) redraw the distribution of different frequency components. [3 *marks*]

(ii) interpret the result in (a) with reference to the new signal formed. [3 *marks*]

2.

(a) Two symmetrical rectangular pulses $s_1(t)$ and $s_2(t)$ have pulse widths τ_1 and τ_2 respectively. Heights of both pulses are 1. If $\tau_1 > \tau_2$,

(i) find the convolution $s(t) = s_1(t) * s_2(t)$. [4 *marks*]

(ii) sketch $s(t)$. [3 *marks*]

(b) (i) Find the Fourier Transform $S_1(\omega)$ of $s_1(t)$. [4 *marks*]

(ii) Find the Fourier Transform $S(\omega)$ of $s(t)$. [4 *marks*]

(iii) Sketch $S_1(\omega)$ and $S(\omega)$ on the same diagram. Compare the two sketches and comment on their differences. [5 *marks*]

3.

(a)

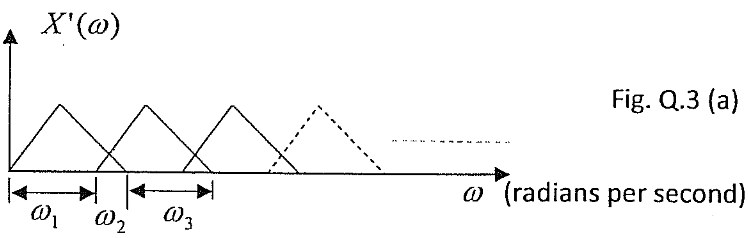


Fig. Q.3 (a)

A signal $x(t)$ is sampled using an impulse train $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$.

The Frequency spectrum of the sampled signal $x'(t) = p(t).x(t)$ is shown in Fig.Q.3 (a)

(i) What is the relationship between ω_1 and ω_3? [2 *marks*]

(ii) What is the relationship between ω_1 and T? [3 *marks*]

(iii) How can $x(t)$ be extracted from $x'(t)$? [3 *marks*]

(iv) Suggest a value for ω_2 so that the signal extracted in (iii) has minimum distortions.

[3*marks*]

(v) If a rectangular pulse train is used instead of the impulse train $p(t)$ to sample $x(t)$, how would you modify the sketch of $X'(\omega)$ vs. ω shown in Fig.Q3(a)? [3 *marks*]

(b)

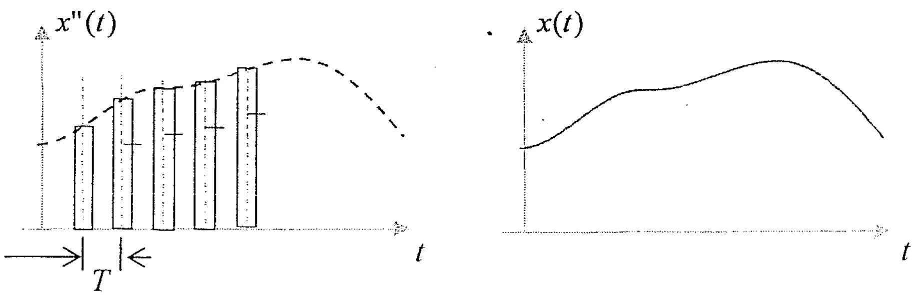
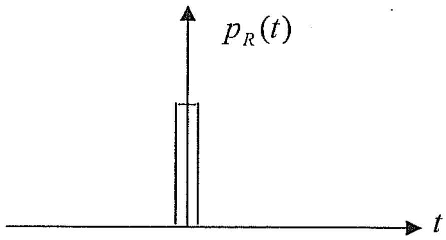


Fig. Q. 3 (b)



The signal $x(t)$ given in (a) is *flat top sampled* using a rectangular pulse $p_R(t)$.

as shown in fig. Q. 3 (b) . The resulting signal is $x''(t)$.

(i) Write the relationship between $x''(t)$, $p_R(t)$, $p(t)$ and $x(t)$. [3 *marks*]

(ii) Write the relationship between $P_R(\omega)$, $X(\omega)$ and $X''(\omega)$. [3 *marks*]

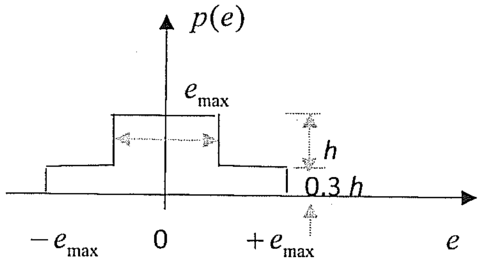
4.

(a) A sinusoidal signal having a peak-peak voltage of 12.8 V is discretized (quantized) into *N* levels. If the discretized signal is *pulse code modulated* into a 7-bit word find

(i) the value of *N*. [3 *marks*]

(ii) maximum possible quantization error ($e_{\max}$). [4 *marks*]

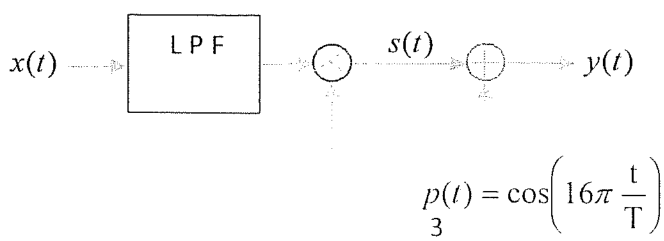
(b) The quantization error (e) of the above process has the probability distribution shown below:



(i) How is noise added to the process? Explain. [5 *marks*]

(ii) Find the signal to noise ratio S/N . [8 *marks*]

5.



LPF – ideal lowpass filter

The impulse train $x(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$ is input to the ideal lowpass filter whose cutoff frequency $f_c = \frac{1.5}{T}$. The filtered signal is multiplied by a cosine wave $p(t)$ to form $s(t)$. Finally $p(t)$ is added to $s(t)$.

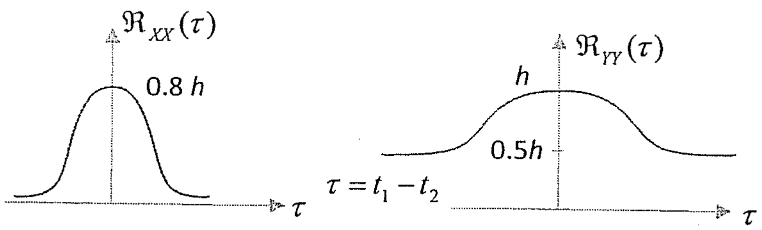
(a) Show that the Fourier transform of $x(t)$ is an impulse train. [Hint: Show that $X(\omega) = \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$, where ω_0 is a constant] [8 *marks*]

(b) Find $y(t)$. [6 *marks*]

(c) Sketch $y(t)$. [6 *marks*]

6.

(a) Autocorrelation functions of two signals *X* and *Y* are shown below:



(i) Which signal is fast varying? *X* or *Y*? Justify your answer. [5 *marks*]

(ii) Find the power of the signal *Y*. [5 *marks*]

(b) (i) What information can be retrieved from the Power Spectral Density function (PSD) of a random process? [4 *marks*]

(ii) The PSD of a noise signal is $S_{xx}(\omega)$. This signal is passed through a lowpass filter whose transfer function is $H(\omega)$. Explain how you would calculate

1. PSD of noise [3 *marks*]
2. noise power [3 *marks*]

at the filter output.

7.

(a) What is Quadrature Amplitude Modulation (QAM)? For 8-QAM explain how the final QAM signal is generated from the given input data. [5 *marks*]

(b) (i) What is Orthogonal Frequency Division Multiplexing (OFDM)? [3 *marks*]

(ii) With the help of a block diagram explain various stages of OFDM. [3 *marks*]

(iii) What is the role of QAM in OFDM? [3 *marks*]

(iv) What are the main advantages of OFDM over other digital transmission standards?

[3 *marks*]

(v) Give one major application of OFDM. [3 *marks*]

8.

(a) (i) What is an optimum filter (Wiener-Hopf filter)? [5 *marks*]

(ii) The total noise N_0 at the receiving Filter output is given by

$$N_0 = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\left|H_{op}(\omega) - \frac{S_m(\omega)}{S_m(\omega)+S_n(\omega)}\right|^2 \frac{S_m(\omega)}{S_m(\omega)+S_n(\omega)} + \frac{S_m(\omega)S_n(\omega)}{S_m(\omega)+S_n(\omega)}\right]d\omega$$

where $S_m(\omega)$ and $S_n(\omega)$ are power spectral densities of the signal and the noise at the input of the receiving filter respectively. $H_{op}(\omega)$ is the transfer function of the optimum filter.

Show that $H_{op}(\omega) = \dfrac{S_m(\omega)}{S_m(\omega)+S_n(\omega)}$. [10 *marks*]

(b) A memoryless source emits messages $m_1, m_2, .. m_n$ with probabilities $p_1, p_2, ... p_n$ respectively. Find the entropy of the source. [5 *marks*]