00027

The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc. / B. Ed Degree Programme

Department	**: Mathematics**
Level	**: 04**
Name of the Examination	**: Final Examination**
Course Title and - Code	**: Continuous Functions – PEU4315**
Academic Year	**: 2023/2024**
Date	**: 23.10.2023**
Time	**: 09.30 a.m. To 11.30 a.m.**
Duration	**: Two Hours.**

General Instructions

- This question paper consists of **(06)** questions on **(02)** pages.
- Answer only **(04)** questions. All questions carry equal marks.

1. (a). (i). Let $E \subseteq \mathbb{R}$, $f: E \to \mathbb{R}$ be a function, and c be a limit point of E and $l \in \mathbb{R}$.

   State the definition of "$\lim_{x \to c} f(x) = l$".

   (ii). Let $f(x) = x^3, x \in \mathbb{R}$. Prove that $\lim_{x \to 2} f(x) \neq 4$.

   (b). Prove that the set of natural numbers, $\mathbb{N}$ has no limit points.

[25 Marks]

2. (a). Let $f(x) = \begin{cases} 1, & x \in [0,1) \\ 2, & x = 1 \\ 3, & x \in (1,2] \end{cases}$.

   (i). Find the value of $\lim_{x \to 1^-} f(x)$. Justify your answer.

   (ii). Find the value of $\lim_{x \to 1^+} f(x)$. Justify your answer.

   (iii). Does the $\lim_{x \to 1} f(x)$ exist? Justify your answer.

   (b). Let $g(x) = x^3 + x$ defined on the interval $(0, 2)$. Prove that $\lim_{x \to 1} g(x) = 2$.

[25 Marks]

3. (a). Let $g(x) = (1-x)\sqrt{x}$ for each $x \in (0,1)$.

   By using the Sandwich theorem, show that $\lim_{x\to 1^-} g(x) = 0$.

   (b). Prove that if $\lim_{x\to\infty} f(x) = l$ exists for some $l \in \mathbb{R}$, then l is unique.

   (c). Using the definition of limit, prove that

$$\lim_{x\to\infty} \frac{x^2+3}{5x^2+3x+16} = \frac{1}{5}.$$

[25 Marks]

4. (a). Let f, g be functions, c_1, c_2 be real numbers such that $(c_1, \infty) \subseteq Domn(f)$, $(c_2, \infty) \subseteq Domn(g)$. Suppose $\lim_{x\to\infty} f(x)$, $\lim_{x\to\infty} g(x)$ exist. Prove that $\lim_{x\to\infty}[f(x)g(x)]$ exists and $\lim_{x\to\infty}[f(x)g(x)] = \lim_{x\to\infty} f(x) \lim_{x\to\infty} g(x)$.

   (b). Let $f(x) = \frac{6x^3+3x^2+4x+2}{35x^3+36x^2+34x+15}$ for each $x > 0$. Show that $\lim_{x\to\infty} f(x) = \frac{6}{35}$.

[25 Marks]

5. (a). Let $g(x) = \frac{x^2+3x+1}{x^2+7}, x \in (1,5)$.

   Prove that $g(x)$ is continuous at 3.

   (b). Let $f(x) = \begin{cases} x^3 + x + 2, & x \geq 3 \\ 15, & x < 3 \end{cases}$.

   (i). Prove that f is right-continuous at 3.

   (ii). Prove that f is not continuous at 3.

[25 Marks]

6. (a). Suppose f is a function defined on an interval I and f is uniformly continuous on I. Prove that f is continuous on I.

   (b). Let $f(x) = 2x + 3, x \in \mathbb{R}$. Show that f is uniformly continuous on $\mathbb{R}$.

[25 Marks]