THE OPEN UNIVERSITY OF SRI LANKA

Bachelor of Technology (Civil) – Level 5

CEX5233 STRUCTURAL ANALYSIS

FINAL EXAMINATION - 2015/2016

Time Allowed – 3 Hours

Date: 27th November 2016 Time: 09.30 – 12.30 Hrs

This paper consists of **EIGHT (8)** questions. Answer **any FIVE (5)** questions.

All questions carry equal marks

QUESTION 1

(i) Generalized Hooke's law with usual notation is given as

$$\sigma_{ij} = C_{ijpq}\epsilon_{pq}$$

where C_{ijpq} is the elastic constants matrix. Show that there are 21 elastic constants for a general three dimensional stress field. (4 Marks)

(ii) Stress tensor for a homogenous, isotropic material can be written with usual notation as

$$\sigma_{ij} = \lambda\epsilon_{pp}\delta_{ij} + 2\mu\epsilon_{ij}$$

where λ and μ are Lame constants. δ_{ij} is the Kronecker delta. Using above expression obtain six independent stress components. (6 Marks)

(iii) Obtain expressions for Young's Modulus (E) and Poisson's ratio (v) in terms of Lame constants (λ, μ). (6 Marks)

(iv) Poisson's ratio can be written with Bulk Modulus (K) and Shear Modulus (G) as

$$v = \frac{(3K - 2G)}{(6K + 2G)}$$

Determine the possible regions for Poisson's ratio. (4 Marks)

QUESTION 2

(i) Airy's stress function (Φ) can be used to find the stress field of two dimensional stress problems with zero body forces. It is written with usual notation as

$$\nabla^4\Phi = \frac{\partial^4\phi}{\partial x^4} + 2\frac{\partial^4\phi}{\partial x^2 y^2} + \frac{\partial^4\phi}{\partial y^4} = 0$$

Write three stress components; σ_{xx}, σ_{yy} and σ_{xy}. (4 Marks)

(ii) A cantilevered beam with length L (breadth and depth are t, d, respectively) is loaded with a concentrated load (P) as shown in Figure Q2.

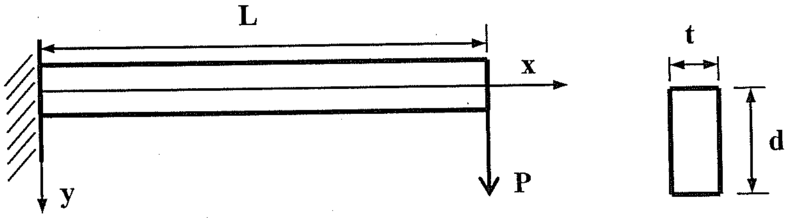


Figure Q2

(a) Show that $\Phi = Ay^3 + Bxy + Cxy^3$ is a possible stress function for the above problem.

(4 Marks)

(b) Obtain the values for A, B and C applying relevant boundary conditions. (6 Marks)

(c) Obtain expressions for stress components; σ_{xx}, σ_{yy} and σ_{xy}. (6 Marks)

QUESTION 3

(i) Briefly explain what do you understand by "**Compatibility Conditions**" in theory of elasticity.
(3 Marks)

(ii) The stress tensor at a point of a particular stress field is given below

$$\sigma_{ij} = \begin{bmatrix} -5 & 2 & -3 \\ 2 & 2 & 1 \\ -3 & 1 & -1 \end{bmatrix} MPa$$

a) Determine six independent stress components. (3 Marks)

b) Determine the stress invariants. (4 Marks)

c) Show that the principal stresses are 2.55, 0.60 and -7.15 MPa. (4 Marks)

d) Determine the directions of principal axes. (6 Marks)

QUESTION 4

(i) Explain why "**Degree of Statical Indeterminacy**" of a structure is important in structural analysis. (4 Marks)

(ii) A continuous beam (ABCD) is shown in Figure Q4. Flexural rigidities of members AB and CD are equal to EI and member BC is 2EI. Uniformly distributed load (w) is acting on member BC and two concentrated loads wl and $2wl$ in members AB and CD, respectively.

a) Determine the degree of statical indeterminacy of the beam. (3 Marks)

b) Draw a released structure. (3 Marks)

c) Determine the flexibility matrix for the drawn released structure. (4 Marks)

d) Determine bending moments at B and C. (6 Marks)

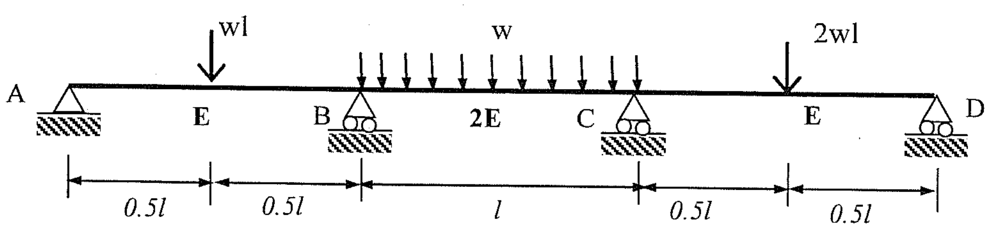


Figure Q4

QUESTION 5

(i) Explain how "**Kinematic Indeterminacy**" of a structure varies from its "**Statical Indeterminacy**". (4 Marks)

(ii) A frame structure shown in Figure Q5. Flexural rigidities of members are same. Find the free nodal displacements at B using the displacement method. You can neglect the axial deformation. (10 Marks)

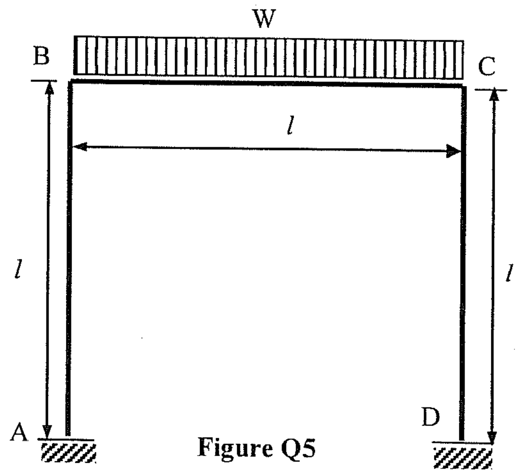


Figure Q5

(iii) Using above results, determine the bending moment at B. (6 Marks)

QUESTION 6

(i) Briefly explain what you understand by the plastic moment of resistance of a beam. (3 Marks)

(ii) A two-bay frame structure is shown in Figure Q6. Dimensions and plastic moments of the beam are given in the figure.

(a) Draw possible failure mechanisms. (3 Marks)

(b) Determine load factors for each failure mechanism. (7 Marks)

(c) Determine the most probable failure mechanism. (4 Marks)

(d) Explain how you can ensure the unique solution. (3 Marks)

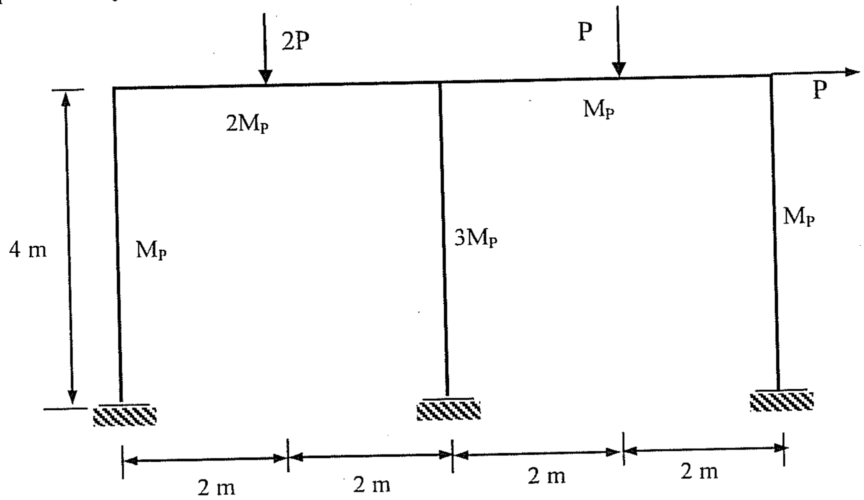


Figure Q6

QUESTION 7

(i) Briefly explain "axisymmetric problems" used in structural analysis. (3 Marks)

(ii) Spherical shell has a radius r as shown in Figure Q7. It is subjected to its self-weight W per meter length. (Note that *w* is a surface load on the shell)

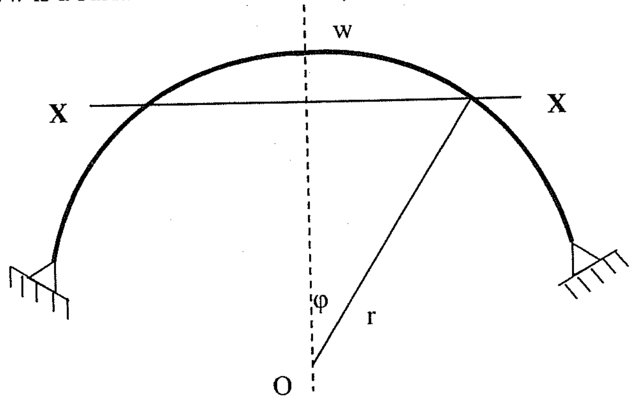


Figure Q7

(a) Determine circumferential and hoop stresses at X-X section level. (8 Marks)

(b) Using the obtained results in section (a), determine the maximum possible value of φ. (4 Marks)

(iii) "**Null method**" and "**Out of balance method**" are used to balance Wheat Stone bridge circuit in experimental stress measurement. Briefly explain the specific use of these two methods. (5 Marks)

QUESTION 8

(i) List three assumptions used in the analysis of thin plates with small deflections. (4 Marks)

(ii) Write strain-displacement relations for a rectangular plate with usual notations. (6 marks)

(iii) A simply supported rectangular plate of sides a and b is carrying a distributed loading as shown in Figure Q8.

$q = q_0 sin\frac{\pi x}{a} in\frac{\pi y}{b}$ where q_0 is a constant.

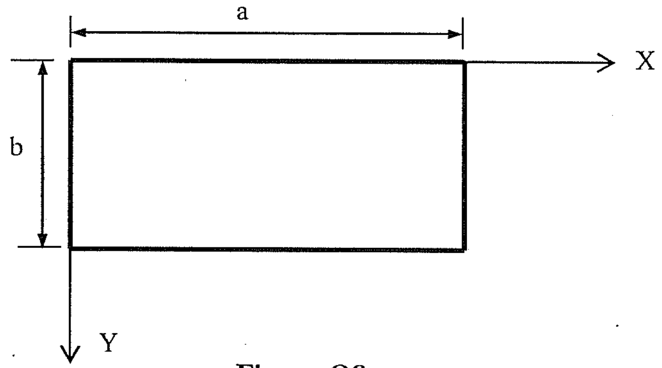


Figure Q8

(a) Show that lateral deflection of the plate is

$$w = \frac{q_0}{\pi^4 D\left(\frac{1}{a^2}+\frac{1}{b^2}\right)} sin\frac{\pi x}{a} sin\frac{\pi y}{b}$$

where D is flexural rigidity of the plate. (7 Marks)

(b) Obtain an expression for the maximum lateral deflection of the plate. (3 marks)

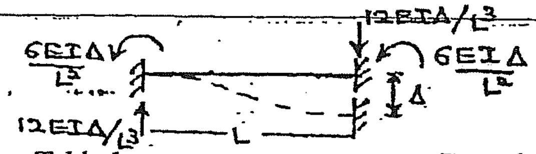
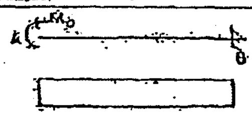
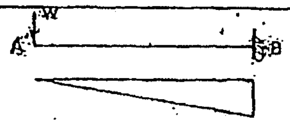
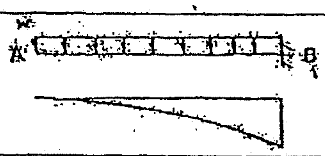
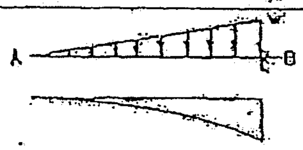
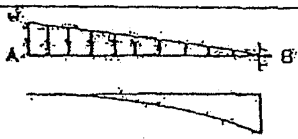
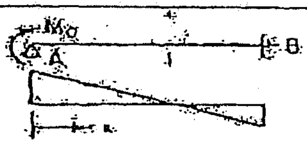
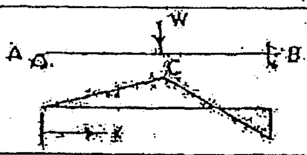
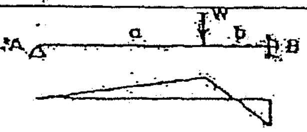
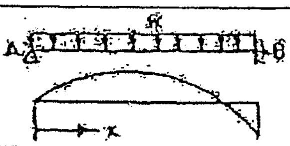
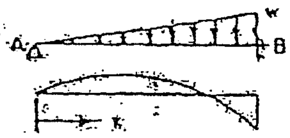
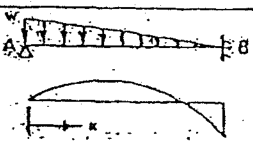
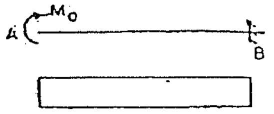
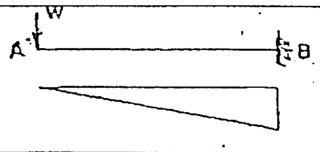
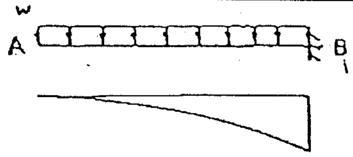
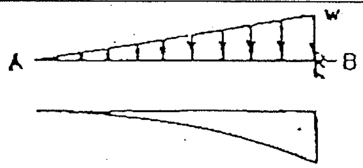
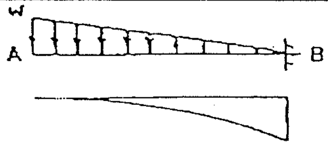


Table 1 Formulas for Beams

Structure	Shear	Moment	Slope	Deflection ↓
Cantilever Beam				
	0	M_o	$\theta_A = \frac{M_o L}{EI}$	$Y_A = \frac{M_o L^2}{2EI}$
	W	$M_B = -WL$	$\theta_A = -\frac{WL^2}{2EI}$	$Y_A = \frac{WL^3}{3EI}$
	$S_B = -WL$	$M_B = -\frac{WL^2}{2}$	$\theta_A = -\frac{WL^3}{6EI}$	$Y_A = \frac{WL^4}{8EI}$
	$S_B = -\frac{WL}{2}$	$M_B = -\frac{WL^2}{6}$	$\theta_A = -\frac{WL^3}{24EI}$	$Y_A = \frac{WL^4}{8EI}$
	$S_B = -\frac{WL}{2}$	$M_B = -\frac{WL^2}{2}$	$\theta_A = -\frac{WL^3}{8EI}$	$Y_A = \frac{11WL^4}{120EI}$
Propped Cantilever				
	$S_A = -\frac{3M_o}{2L}$	$M_B = -\frac{M_o}{2}$	$\theta_A = -\frac{M_o L}{4EI}$	$Y_{max} = \frac{M_o L^2}{27EI}$ at $x = \frac{L}{3}$
	$S_A = -\frac{3M_o}{2L}$	$M_B = -\frac{3WL}{16}$ $M_c = \frac{5WL}{32}$	$\theta_A = \frac{WL^2}{32EI}$	$Y_{max} = 0.00962\frac{WL}{EI}$ at $x = 0.447$
	$S_A = \frac{Wb^2}{2L^3}(a+2L)$ $S_B = -\frac{Wa}{2L^3}(3L^2 - a^2)$	$M_B = -\frac{Wab}{L^2}(a+\frac{b}{2})$	$\theta_A = \frac{Wab^2}{4EIL}$	$Yo = \frac{Wa^2b^2}{12EIL^3}(3L$
	$S_A = +\frac{3WL}{8}$	$M_B = -\frac{WL^2}{8}$	$\theta_A = \frac{WL^3}{48EI}$	$Y_{max} = 0.0054\frac{WL}{EI}$ at $x = 0.42$
	$S_A = +\frac{WL}{10}$	$M_{max} = 0.03WL^2$ at $x = 0.447L$ $M_B = -\frac{WL^2}{15}$	$\theta_A = \frac{WL^3}{120EI}$	$Y_{max} = 0.00239\frac{W}{EI}$ at $x = 0.44$
	$S_A = +\frac{11WL}{40}$	$M_{max} = 0.0423WL^2$ at $x = 0.329L$ $M_B = -\frac{7WL^2}{120}$	$\theta_A = \frac{WL^3}{80EI}$	$Y_{max} = 0.00305\frac{W}{EI}$ at $x = 0.40$

Structure	Shear	Moment	Slope	Deflection
Cantilever Beam				
	0 –	M_o	$\theta_A = \frac{M_oL}{EI}$	$Y_A = \frac{M_oL^2}{2EI}$
	W	M_B = -WL	$\theta_A = -\frac{WL^2}{2EI}$	$Y_A = \frac{WL^3}{3EI}$
	S_B= -WL	$M_B = -\frac{WL^2}{2}$	$\theta_A = -\frac{WL^3}{6EI}$	$Y_A = \frac{WL^4}{8EI}$
	$S_B = -\frac{WL}{2}$	$M_B = -\frac{WL^2}{6}$	$\theta_A = -\frac{WL^3}{24EI}$	$Y_A = \frac{WL^4}{8EI}$
	$S_B = -\frac{WL}{2}$	$M_B = -\frac{WL^2}{2}$	$\theta_A = -\frac{WL^3}{8EI}$	$Y_A = \frac{11WL^4}{120EI}$
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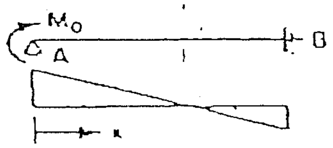
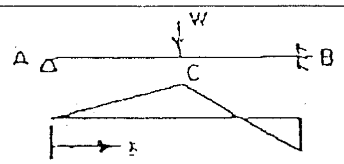
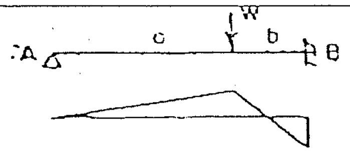
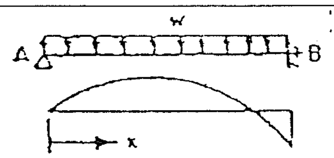
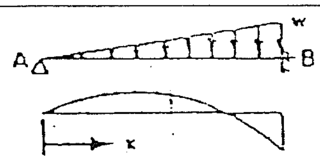
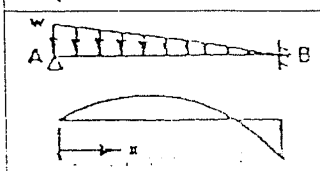
	$S_A = -\frac{3M_o}{2L}$	$M_B = -\frac{M_o}{2}$	$\theta_A = -\frac{M_oL}{4EI}$	$Y_{max} = \frac{M_oL^2}{27EI}$ at $x = \frac{L}{3}$
	$S_A = -\frac{5W}{16}$	$M_B = -\frac{3WL}{16}$ $M_c = \frac{5WL}{32}$	$\theta_A = \frac{WL^2}{32EI}$	$Y_{max} = 0.00932\frac{WL^3}{EI}$ at $x = 0.447L$
	$S_A = \frac{Wb^2}{2L^3}(a+2L)$ $S_B = -\frac{Wa}{2L^3}(3L^2-a^2)$	$M_B = -\frac{Wab}{L^2}(a+\frac{b}{2})$	$\theta_A = \frac{Wab^2}{4EIL}$	$Y_o = \frac{Wa^2b^3}{12EIL^3}(3L+a)$
	$S_A = \frac{3WL}{8}$	$M_B = -\frac{WL^2}{8}$	$\theta_A = \frac{WL^3}{48EI}$	$Y_{max} = 0.0054\frac{WL^4}{EI}$ at $x = 0.422L$
	$S_A = \frac{WL}{10}$	$M_{max} = 0.03WL^2$ at $x = 0.447L$ $M_B = -\frac{WL^2}{15}$	$\theta_A = \frac{WL^3}{120EI}$	$Y_{max} = 0.00239\frac{WL^4}{EI}$ at $x = 0.447L$
	$S_A = \frac{11WL}{40}$	$M_{max} = 0.0423WL^2$ at $x = 0.329L$ $M_B = -\frac{7WL^2}{120}$	$\theta_A = \frac{WL^3}{80EI}$	$Y_{max} = 0.00305\frac{WL^4}{EI}$ at $x = 0.402L$