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THE OPEN UNIVERSITY OF SRI LANKA

Department of Civil Engineering

Bachelor of Technology - Level 5

CEX5231 - MECHANICS OF FLUIDS

FINAL EXAMINATION 2015/2016

Time Allowed: Three Hours

Date: 06th December, 2016 Time: 0930 – 1230 hrs

ANSWER **ALL THREE** QUESTIONS IN PART A AND **ANY TWO** QUESTIONS IN PART B. ALL QUESTIONS CARRY EQUAL MARKS.

PART A

Answer **all three** questions in this section.

1) The power required to overcome the resistance of air to the movement of a vehicle at a design velocity of 100 km/hour is to be studied by conducting a physical model study with air as the working fluid. The study is to use a model scale of 1:10 with an air velocity of 10 ms^{-1}.
The density of air is 1.1 kgm^{-3} and the dynamic viscosity of air is 2×10^{-5} Pas. The prototype vehicle has a length of 6 m, a width of 2.5 m and a height of 2 m.
It is expected that the Drag force on the vehicle, F, is a function of the velocity of the vehicle, V, the length of the vehicle, L, the height of the vehicle, H, the width of the vehicle, W, the density of air, ρ and the viscosity of air, μ.

a) What does Buckingham's Π - theorem say about the relationship between the number of physical variables describing a problem, the number of dimensions contained in these variables and the number of non - dimensional variables needed to describe the problem?
b) Derive a relationship between the non – dimensional drag force on the vehicle and other non – dimensional quantities.
c) Is the selected air velocity in the model suitable? Explain your answer.
d) The Drag Force on the model is found to be 2 N. Calculate the power required to overcome air resistance when the prototype is moving at the design speed.

2)

a) An equation used in the computation of flow profiles in open channel flows is $\frac{dy}{dx} = \frac{S_0 - S_f}{1 - Fr^2}$.
   i) Identify the variables $\frac{dy}{dx}$, S_0, S_f and Fr in this equation using a neat diagram.
   ii) List the important assumptions that have been made to derive this equation.
   iii) Explain why this equation can not be used when the flow depth is close to the critical depth.

(Q2 continued...)

b) A wide rectangular channel has two sections AB and BC where there is a change in bed slope. The section AB has a slope of 0.0004 while the section BC has a slope of 0.01. The Manning's coefficient of both sections is 0.01.The channel carries a steady discharge of 20 m^3s^{-1} per metre width of the channel. The flow far upstream and far down stream of B is observed to be uniform.

   i) Calculate the critical depth for the given channel and discharge.

   ii) Calculate the uniform depths for the two sections of the channel for the given discharge.

   iii) Determine whether the two sections are having a mild slope or a steep slope under the given conditions.

   iv) What is the depth at the point where the slope changes? Explain your answer.

   v) Sketch and classify the surface profiles (from M1, M2, M3, S1, S2, S3, C1, C2, C3 etc.) in sections AB and BC.

3)

a) An open channel is considered wide if it has a width much greater than the flow depth. Show that for uniform flow in a wide channel the bottom shear stress, τ_0 is given by the equation; $\tau_0 = \rho g h S_0$, where h is the depth of flow and S_0 is the channel slope.

b) Use the Manning's equation to show that the bottom shear stress in a wide open channel is related to the discharge, Q, by the equation $\tau_0 = \rho g \left(\frac{Qn}{b}\right)^{\frac{3}{5}} S_0^{\frac{7}{10}}$ when the flow is uniform. Here b is the width of the channel and n the Manning's coefficient.

The Meyer – Peter – Muller formula for bed load transport is given by
$\frac{q_s \rho^{1/2}}{\{\rho(s-1)gd\}^{3/2}} = 8\left\{\frac{\rho u_*^2}{\rho(s-1)gd} - 0.047\right\}$ where the shear velocity $u_* = \sqrt{\frac{\tau_0}{\rho}}$, q_s is the rate of sediment transport, ρ is the density of water, s the specific gravity of sediment, d the grain diameter.

c) State the dimensions and the units of the rate of sediment transport q_s.

A long straight river is 100 m wide, has a uniform cross section and has a slope of 0.0001. The river bottom consists of fine sand of diameter 0.1 mm and specific gravity 2.65.

d) Use the Meyer – Peter – Muller formula and the results of the section b) to estimate the discharge for which the sediment just begins to move ($q_s = 0$).

e) Calculate the rate of sediment transport in the river when the discharge is twice the value estimated in section d).

PART B

Answer **any two** questions in this section.

4)

a) A long open channel has a uniform rectangular cross section of width b as shown in Figure Q4(a).

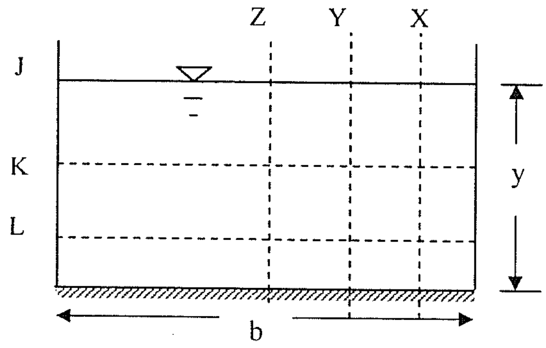

Figure Q4(a)

i) Draw the velocity distribution in the vertical plane of the channel (at X, Y and Z). Explain your answer.
ii) Mark the maximum velocity in each profile given in (a). Explain your answer.
iii) Draw the velocity distribution in the horizontal plane of the channel (at J, K and L). Explain your answer.

b) An irrigation channel has a rectangular cross section of width 1 m. The channel carries a discharge of 0.5 m^3s^{-1} at an average velocity of 1 ms^{-1}.

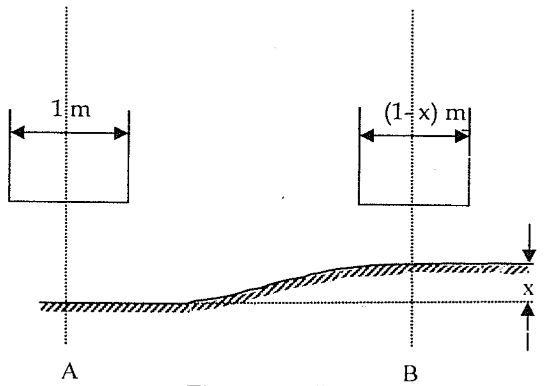

Figure Q4 (b)

i) What is the definition of critical depth in open channel flow?
ii) Show that the flow depth, y, critical depth, y_c and the specific energy, E in an open channel of rectangular cross section are related by the expression $\frac{E}{y_c} = \frac{y}{y_c} + \frac{1}{2}\left(\frac{y}{y_c}\right)^{-2}$

The channel has a smooth transition where the width is reduced by x m and the bed level is raised by a smooth step of height x m, as shown in the Figure Q4(b).

iii) Calculate the depth of flow in the channel after the transition for x = 0.05 m.
iv) Indicate the flow conditions at A and B on a graph of y/y_c against E/y_c.
v) Calculate the magnitude and direction of the force on the transition when x = 0.05 m.

5) Water is pumped from a reservoir X, to a reservoir Y, through a main pipe line AB of 0.45 m diameter and 1400 m length as shown in Figure Q5. The elevations of the water levels in reservoirs X and Y are 18 m and 30 m respectively. The pump is located near the low level reservoir. At a point (D) along the main line, at a distance of 450 m from the reservoir Y (BD = 450 m), a branch line of 0.3 m diameter and 360 m length takes off to discharge 180 litre/sec to the atmosphere, at an elevation of 25.5 m. The friction factor for both pipes can be taken as 0.032.

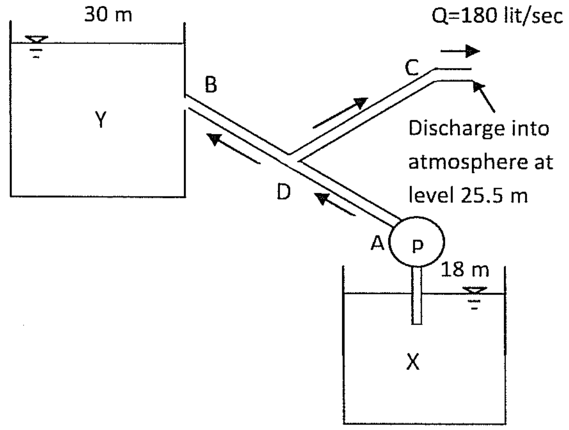

Figure Q5

a) Determine the flow rate into the reservoir Y.
b) Calculate the theoretical power of the pump assuming the delivery valve as 20 m.

6)

An air flow with a uniform velocity **U** approaches a long, smooth cylinder of radius **a** as shown in Figure Q6.

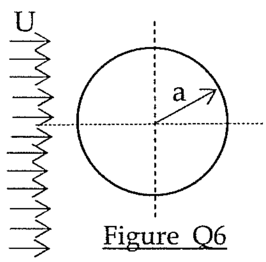

Figure Q6

a) Sketch on separate diagrams, the stream lines of the flow past the cylinder for the following cases and explain the differences using neat diagrams.
   i) When the fluid is an ideal fluid
   ii) When the fluid is a real fluid and the Reynolds number is very low.
   iii) When the fluid is a real fluid and the Reynolds number is high.

(Q6 continued....)

b) Using neat figures of velocity profiles, explain what you understand by the term "fully developed boundary layer".

c) Explain using neat figures of velocity profiles, the difference between laminar and turbulent boundary layers on a flat plate.

d) Explain the differences in shear stress and boundary layer thickness for the two types of boundary layer in c).

e) Explain the physical significance of the displacement thickness and the momentum thickness.

7)

The Navier-Stokes equations can be written in the form

$$\frac{\partial \underline{q}}{\partial t}+(\underline{q}.\nabla)\underline{q}=-\frac{1}{\rho}\underline{\nabla}(p^*)+\frac{\mu}{\rho}\nabla^2\underline{q} \quad (1) \quad \text{and} \quad \nabla.\underline{q}=0 \quad (2)$$

where $\underline{q}=u\underline{i}+v\underline{j}+w\underline{k}$ is the velocity vector and $p^*=p+\rho gz$ is the piezometric pressure with z measured in the $\underline{k}$ direction.

a) List the assumptions that have been made in the derivation of equations (1) and (2) .
b) Explain briefly how equation 1) has been derived.
c) Explain the physical significance of the second term in the left hand side of equation (1).

A fluid of density $\boldsymbol{\rho}$ and dynamic viscosity $\boldsymbol{\mu}$ flows in a wide open channel that has a small angle of slope $\boldsymbol{\theta}$, as shown in Figure 2. The flow is steady, two-dimensional and uniform in the x direction . The depth of the flow is **h** . The flow is laminar.

d) Apply the -force momentum relation to the elemental control volume shown in Figure 7 and obtain a relationship between the gradient of the shear stress $d\tau/dy$ and the slope of the channel. State all assumptions and explain your answer.

e) Integrate the relationship obtained in section d) and obtain expressions for the velocity profile u(y) and the discharge per unit width in the channel. State all boundary conditions and explain your answer.

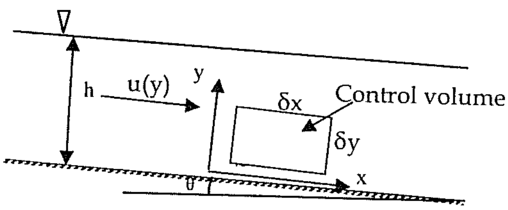

Figure 7

8)
A long, straight river, which has a width of 200 m, flows in the middle of a long, straight area of deposited sediment of 3 km width - that is bounded below and on both sides by impervious rock as shown in Figure Q8. The deposited sediment is homogenous and isotropic and has a permeability of 2.5 m/day. The depth of sediment below the river bed is 25 m. The area is recharged at an average rate of **q** mm/day.
The water level in the river is maintained at a constant level of 5 m above the river bed by a dam and the water level in the ground on both sides of the river is at a steady state. There is a well located 500 m away from the bank of the river, as shown in the figure. The water level in the well is found to be 0.2 m above the water level in the river.

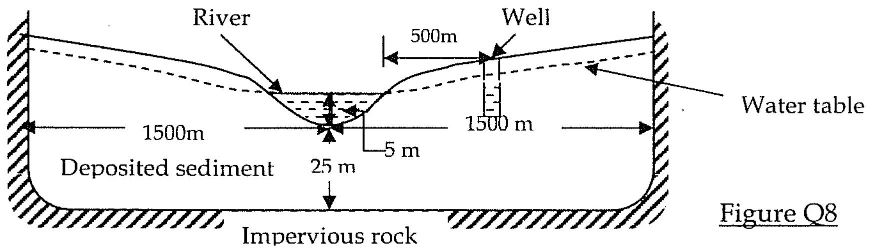

Figure Q8

a) Derive, from first principles, a differential equation governing the variation of the groundwater level on one side of the river. State all your assumptions.

b) Sketch the streamlines of groundwater flow and identify the areas where the assumptions made in section a) will not be correct.

c) Estimate the average rate of recharge due to the infiltration of rainfall into the aquifer. State all your assumptions.

An industry begins to extract water from the well at a constant daily rate. A new steady state is reached due to the pumping and the water level in the well is now observed to the 0.1 m above the water level in the river.

d) Sketch the variation of the groundwater level with distance from the river that you would expect under the new steady state. Explain your answer.

After some time the pumping rate is increased and the water level in the well is found to be steady at a level 0.1 m below the water level in the river.

e) Sketch the variation of the groundwater level with distance from the river that you would expect under increased rate of pumping. Explain your answer.

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