The Open University of Sri Lanka

B.Sc. Degree Programme Level 05

Department of Computer Science

CSU 5304 – Mathematics for Computing

Final Examination - 2023/24

Duration: Two Hours Only (2 Hours)

Date: 07th October 2023   Time: 9.30 a.m. – 11.30 a.m.

Answer Four Questions only.

(01) (i) Define the term **"Function"** in the context of Mathematics.

(ii) Given below are coordinates of a function / graph.

$(45.6 , 65.5) , (48.2 , 68.0) , (41.8 , 62.2) , (46.0 , 66.0) , (50.4 , 70.0)$

    (a) Write the two sets: domain and image.
    (b) Give any set that you think is a suitable **codomain** for the above function / graph.

(iii) Given that

$$g(t) = \begin{cases} 3t^2 + 4 & if\ t \le -4 \\ 10 & if -4 < t \le 15 \\ 1 - 6t & if\ t > 15 \end{cases}$$

Using the above information evaluate each of the following functions.

    (a) $g(-6)$,   (b) $g(1)$ and   (c) $g(21)$

(iv) The functions $f\ and\ g$ are defined such that:

$f : \mathbb{R} \longrightarrow \mathbb{R}$ and $g : \mathbb{R} \longrightarrow \mathbb{R}$

$f : t \longmapsto 4t - 1$ $g : x \longmapsto kx^2$ where k is a constant.

    (a) Find the composite function $f \circ g\ (x)$
    (b) Given that $f \circ g(2) = 12$, what is the value of k.

(25 marks)

(02) (i) Write down the conditions to be satisfied in order to perform the following matrix operations.

    (a) Addition of two matrices

    (b) Multiplication of two matrices

    (c) Finding the inverse of a matrix.

(ii) The two matrices A and B are defined by:

$$A = \begin{bmatrix} -2 & 1 & 2 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 1 & 2 \\ k+3 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix} \text{ where } k \in \mathbb{R}$$

    (a) Obtain the matrix $2A^T - B$; where A^T is the transpose of A.

    (b) (i) Find and simplify an expression for the determinant of B.

        (ii) If B^{-1} does not exist, find the value of k.

(iii) For the matrix $C = \begin{bmatrix} 2 & 2\alpha - \beta & -1 \\ 3\alpha - 2\beta & 4 & 3 \\ -1 & 3 & 2 \end{bmatrix}$ obtain the values of α and β such that

$$C^T = \begin{bmatrix} 2 & -5 & -1 \\ -1 & 4 & 3 \\ -1 & 3 & 2 \end{bmatrix} \text{ where } C^T \text{ is the transpose of matrix } C.$$

(iv) Given that $D = \begin{bmatrix} 0 & 1 & -1 \\ 4 & 3 & -4 \\ 3 & 3 & -4 \end{bmatrix}$, find D^2.

    Hence show that $D^{-1} = D$

(v) Find the determinant of $\begin{bmatrix} 2 & -3 & 5 \\ -3 & 6 & 2 \\ 1 & -2 & 5 \end{bmatrix}$ by expanding along the 1st row only.

$$3 \times 3$$

Note: **(If you select another row or a column marks will not be given)**

(25 Marks)

(03) (i) If a proposition $P(n)$ is given, where $n \in \mathbb{N}$. To prove $P(n)$ is true for all $n \in \mathbb{N}$, suggest a name of a method.

(ii) For the method you suggested, if there are any conditions, that should be satisfied write down in point form.

(iii) Using the method you gave as the answer to (3) part (i) prove the following.

(a) $1^2 + 2^2 + 3^2 + \cdots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$

(b) Verify that for any $n \in \mathbb{N}$, $(6^n - 1)$ is divisible by 5.

(25 Marks)

(04) (i) Give the interval notation to represent the sets given below.

(a) The set of all numbers from 2 through 7 inclusive.
(b) The set of all numbers greater than or equal to 5 and less than 12.
(c) The set of all real numbers greater than -6 and less than or equal to 14.

(ii) Give the inequality notation or set builder notation to describe the following sets.

(a) $\{-3, -2, -1, 0, 1, 2\}$
(b) $\{1, 2, 3, 4\}$
(c) $\{0, 1, 2, 3\}$

(iii) Prove the following identities, stating carefully the set laws you are using at each stage of the proof.

(a) $B \cup (\emptyset \cap A) = B$
(b) $(A \cap B) \cup (A \cap B') = A$

(iv) In a group of 60 people 27 like to drink tea and 42 like to drink coffee, and each person likes at least one of the two drinks. How many like both tea and coffee?

(25 Marks)

(05) (i) (a) If p and q are two statements, the implication is defined by $p \longrightarrow q$ (*if p then q*).

Give three operations that are related to this implication.

(b) An implication is given below.

" If it is raining then there are clouds in the sky "

According to the answer you gave to (i)(a), write the three operations.

(ii) (a) Construct a truth table for $(p \longrightarrow q) \vee (q \longrightarrow p)$ and show that the formula is always true.

(b) Compute the truth table for NOR logic gate.

(iii) Draw symbols for the logic gates given below:

(a)NAND , (b) XOR , (c) AND and (d) NOT   (25 Marks)

(06)(i) Using logical equivalences or laws, prove the following algebraically.

$\neg q \lor [\neg(p \lor \neg p) \land r] \longrightarrow s \equiv s \lor q$

(ii) Find the truth values of the following formula in predicate logic

(a)$\forall x\, P(x) \longleftrightarrow \neg\exists x\, \neg P(x)$
(b)$\exists x\, P(x) \longleftrightarrow \neg\forall x\, P(x)$
(c)$\forall x\exists y\, P(x,y) \longleftrightarrow \exists y\forall x\, P(x,y)$

(iii) Translate each of the following statements into logical expressions using predicates, quantifiers, and logical connectives.

Predicates are given below.

$C(x)$; x is a CSU260 student.

$L(x)$; x loves music.

The universe of discourse for the variable x is all students.

(a) Every student love music.
(b) No students love music.
(c) Some students love music.
(d) Every CSU260 student loves music.
(e) Some CSU260 students love music.

(25 Marks)

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