The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc. / B. Ed. Degree Programme

Department	: Mathematics
Level	: 05
Name of the Examination	: Final Examination
Course Title and - Course Code	: Combinatorics–PEU5302
Academic Year	: 2023/24
Date	: 13.10.2023
Time	: 2.00 p.m. To 4.00 p.m.
Duration	: Two Hours.

General Instructions

1. Read all instructions carefully before answering the questions.

2. This question paper consists of **(6)** questions in **(4)** pages.

3. Answer any **(4)** questions only. All questions carry equal marks.

4. Answer for each question should commence from a new page.

5. Draw fully and clearly labelled diagrams where necessary

6. Involvement in any activity that is considered as an exam offense will lead to punishment.

7. Use blue or black ink to answer the questions.

8. Clearly state your index number in your answer script

(01) (a) A committee of 7 members has to be chosen from 6 artists, 4 singers and 5 writers. In how many ways can this be done if the committee consists of at least 3 artists, at least one singer, and at least one writer.

(b) A box contains two white balls, three black balls and four red balls. In how many ways can three balls be drawn from the box, if at least one black ball is to be included in the draw?

(c) How many triangles can be formed by joining 15 points on the plane, in which no line joining any three points? Justify your answer.

(d) Find the number of strings of 4 letters that can be formed with letters of the word EXAMINATION?

(02) (a) How many batting orders are possible, for a cricket team of 11 players, if there is a squad of 16 players, so that player A and player B are always included?

(b) Silva and his wife both have five friends each. Silva has 2 male friends and 3 female friends. His wife has 3 male friends and 2 female friends. In how many different ways can they invite 2 male friends and 2 female friends such that two of them are Silva's friend and two are his wife's friend.

(c) Show that if 5 integers are selected from the first 8 positive integers, then there must be a pair of those integers with sum equal to 7.

(03) (a) How many ways that Computer Science department can select a student from first years, second years, third years or fourth years for a student club if there are 10 students from first year, 8 students from second year, 6 from third year and 4 from final year.

(b) Find the sum of the coefficients of the polynomial $\left(2\sqrt{5} - 2\sqrt{5}\, x + \frac{5}{\sqrt{5}} x^2 - \frac{1}{\sqrt{5}} x^3\right)^{10}$.

(c) (i) A bag contains 3 red balls and 7 black balls. Two balls are drawn at random without replacement. If the second ball is red, what is the probability that the first ball is also red?

(ii) Two dice are rolled. It is known that at least one of the dice always shows 4. Find the probability that the numbers appeared on the dice have a sum 8.

(04) (a) Prove that the sum of the first n non-zero even number is $n^2 + n$.

(b) By using mathematical induction, prove that, each of the following equation is true for all the positive integers.

(i) $(n + 1)^2 + (n + 2)^2 + (n + 3)^2 + \cdots + (2n)^2 = \frac{n(2n+1)(7n+1)}{6}$,

(ii) $(\cos\theta + i\,\sin\theta)^n = (\cos n\theta + i\,\sin n\theta)$,

(c) Show that $2^{2n} - 1$ is divisible by 3.

(05) (a) Let n and r be two positive integers such that $1 \le r \le n$. Use a combinatorial argument to prove Pascal's identity, $^{n+1}C_r = {}^nC_r + {}^n C_{r-1}$,

Hence, deduce that $^{n+2}C_{r+2} = {}^nC_{r+2} + 2\,{}^nC_{r+1} + {}^nC_r$.

(b) Let n and r be two non-negative integers such that $0 \le r \le n$.
Let $c_r \left[\binom{n}{r}\right]$ be the coefficient of the $(r + 1)^{th}$ term of a binomial expansion of $(1 + x)^n$.
Prove that $c_0 + 3c_1 + 5c_2 + \cdots + (2n + 1)c_n = (n + 1)2^n$.

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(c) Let n, m and r be three positive integers such that $1 \leq r \leq n$ and $1 \leq r \leq m$.

Prove each of the following statements using ONLY the combinatorial arguments:

    (i) $^{n}C_2 + {}^{m}C_2 + nm = {}^{n+m}C_2,$

    (ii) $n \times {}^{n-1}C_{r-1} = r \times {}^{n}C_r.$

(06)   (a) Determine the total number of terms in the expansion of $(1 + x + y).^{10}$

    (b) Find the multinomial coefficient of each of the following terms of the expansions

of the expressions given below:

    (i) y^5 in the expansion of $(l - my + ny^2)^6(l + my - ny^2)^6,$

    (ii) x^5 in the expansion of $(2 - x + 3x^2)^6.$

(c) Are the coefficients of xyz^2w^4 and x^2yzw^4 in the expansion of
$(x - 2y - z + w)^8$ the same? Justify your answer.

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