

The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc. / B. Ed. Degree Programme



Department	: Mathematics
Level	: 03
Name of the Examination	: Final Examination
Course Title and - Code	: Applied Calculus I – ADE 3200
Academic Year	: 2023/24
Date	: 26.10.2023
Time	: 09.30a.m. –11.30a.m.
Duration	: Two Hours

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of (6) questions in (2) pages.
3. Answer any (4) questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Involvement in any activity that is considered as an exam offense will lead to punishment.
6. Use blue or black ink to answer the questions.
7. Clearly state your index number in your answer script.

Answer any **FOUR** questions only

1. Determine whether the following statements are **TRUE** or **FALSE**. In each case justify your answer:

- a) The domain of the function defined by $f(x) = \frac{3}{\sqrt{2x-3}}$, is $[\frac{3}{2}, \infty)$. (05 marks)
- b) A discontinuity of a function at a point is where left or right limit does not exist. (05 marks)
- c) The sequence $\{2^n\}_{n=0}^{\infty}$ is a bounded sequence. (05 marks)
- d) The function defined by $f(x) = x^{1/3}$ is a differentiable function in \mathbb{R} . (05 marks)
- e) If $y = \sqrt{1+x}$ then $2(1+x)\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$. (05 marks)

[Total marks 25]

2. Consider the function defined by $f(x) = 1 + 6x^2 - 2x^3$.

- a) Find the domain of f . (02 marks)
- b) Determine the y -intercept(s). (02 marks)
- c) Find $f'(x)$, and hence, determine the interval(s), where f is increasing and decreasing.
Identify the coordinates of local maxima or/and minima and determine their values if exist. (06 marks)
- d) Find $f''(x)$, and hence, identify the interval(s) (if any) that the function f is concave up or/and concave down. If there are any inflection points identify their coordinates. (05 marks)
- e) Find $\lim_{x \rightarrow \infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$? (02 marks)
- f) Sketch the graph of f . (08 marks)

[Total marks 25]

3. Find the answers for the following questions:

- a) Find the equation of the tangent line drawn to $f(x) = 7e^x + 3$ at the point $(0, 10)$. (05 marks)
- b) Using the derivatives rules, calculate $g'(x)$, where $g(x) = \sqrt{1 + \sqrt{1 + \sqrt{x}}}$. (10 marks)
- c) Using implicit differentiation, find $\frac{dy}{dx}$, where $x^4 + 1 - x^2y = 3xy$. (10 marks)

[Total marks 25]

[Turn over

4. a) A water tank is in the shape of a right circular cone with the axis vertical and vertex downward. The tank has radius 3 m and is 5 m high. Initially the tank is filled by water, but at time $t = 0$ (in seconds), a small hole at the vertex is opened and the water starts to drain. When the height of water in the tank has dropped to 3 m, the water is flowing out at $2 \text{ m}^3/\text{s}$. At what rate, in meters per second, is the water level in the tank dropping then?

(Note: The volume of a right circular cone is given by $\frac{1}{3}\pi r^2 h$, where r is the radius and h is the height.) (15 marks)

- b) For the the piecewise function f defined below, find the value of k such that the function is continuous at $x = 2$:

$$f(x) = \begin{cases} \frac{x^3 + x^2 - 16x + 20}{(x-2)^2} & \text{for } x \neq 2 \\ k & \text{for } x = 2 \end{cases}$$

(10 marks)

[Total marks 25]

5. a) State whether $\{(-1)^n\}_{n=0}^{\infty}$ converges or diverges. Explain your answer. (05 marks)

- b) Consider the increasing sequence $\{a_n\}_{n=1}^{\infty}$ with $a_1 = \sqrt{2}$ and $a_{n+1} = \sqrt{2 + a_n}$ for $n \in \mathbb{N}$.

i) Show that $a_n \leq 2$ for all $n \in \mathbb{N}$. (10 marks)

ii) Conclude that the sequence is convergent and write the limit. (10 marks)

[Total marks 25]

6. a) Use comparison test to show that the series $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ is convergent. (09 marks)

- b) For each of the following, determine whether the series converges or diverges:

i) $\sum_{n=1}^{\infty} \frac{(n-1)!}{(n+1)^2}$ (08 marks)

ii) $\sum_{n=0}^{\infty} \left(\frac{n}{3n+1}\right)^n$ (08 marks)

[Total marks 25]

***** END OF QUESTION PAPER *****