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The Open University of Sri Lanka
Faculty of Engineering Technology

Study Programme	: Bachelor of Technology Honours in Engineering
Name of the Examination	: Final Examination
Course Code and Title	: **MEX 6278 – FLUID MECHANICS**
Academic Year	: 2016/17
Date	: 09th, December 2017
Time	: 9.30am -12.30pm
Duration	: 3 hours

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of 8 questions. All questions carry equalmarks.
3. Answer **any 5** questions only.
4. Take acceleration due to gravity and the density of water as**9.81 N/kg** and **1000 kg/m³** respectively where necessary.

Q1.

a) By considering a differential fluid element, show that the vector form of the continuity equation for a compressible, unsteady fluid flow with usual notation is given by,

$$\frac{d\rho}{dt} + \nabla(\rho u) = 0$$

6-marks

Where $\nabla = \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k\right)$

b) Obtain the expressions for the continuity equations of following fluid flows.

i. Steady compressible.

ii. Steady incompressible.

4-marks

c) The velocity components in x and y direction of an incompressible, steady-flow field are $\mathbf{u=x^2+y^2+z^2}$ and $\mathbf{v=xy+yz+z}$ respectively. Determine the velocity component in the z direction (w), required to satisfy the law of conservation of mass.

10-marks

Q2.

(a). By considering velocity variation that causes rotation and angular deformation in a two dimensional differential fluid element shown in Figure Q2, Show that the angular velocity of the fluid flow is given by,

10-marks

$$\omega = \frac{1}{2}\nabla \times V$$

where $\omega = \omega_x\hat{i} + \omega_y\hat{j} + \omega_z\hat{k}$ and $V = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$

10-marks

(b). A velocity field in a flow is given by **V**=2xy**i** +4tz^2**j**-yz**k** ms^{-1}. Find the acceleration, the angular velocity about z-axis, and the vorticity vector at the point **(2m, -1m, 1m)** at **t=2 sec.**

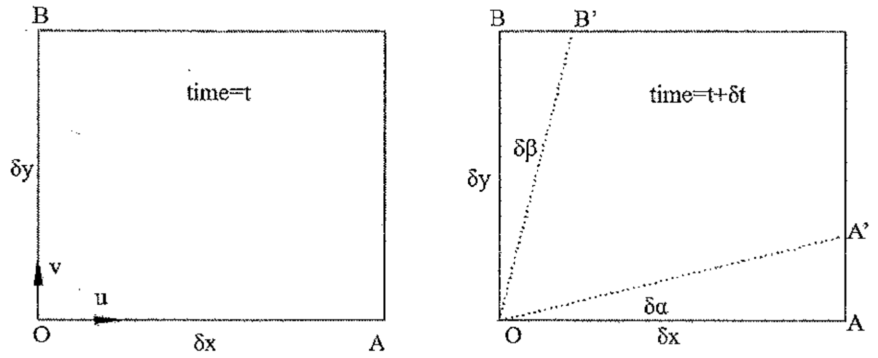

Figure Q2.

Q3.

The differential pressure (***dP***) of a forced vortex motion in a cylindrical container, with usual notation, is given by,

$$dP = \rho r\omega^2 dr - \rho g dz$$

An open cylindrical container **0.5m** in diameter and **0.8m** in height, filled with oil up to **0.5m** and is rotating about its vertical axis. The specific gravity of liquid is **0.88.**

6-marks

(a). Show that the water surface takes the form of a paraboloid.

(b). Calculate the speed at which the liquid will start to spill over.

8-marks

(c). Calculate the speed at which the point of the bottom centre will just expose?

6-marks

Q4.

(a). State the Buckingham's π theorem. **2-marks**

(b). The pressure drop (Δp) across a gate valve depends on the gate opening h, overall depth d, velocity V, density of fluid ρ, viscosity of the fluid μ.

Using Buckingham's π theorem, show that the variables can be expressed as, **8-marks**

$$\Delta p = \rho V^2 f\left(\frac{h}{d}, \frac{\rho V d}{\mu}\right)$$

(c). What is dynamic similarity? **2-marks**

(d). A prototype gate valve which will control the flow in a pipe system that conveys paraffin is to be studied in a model. A **1:5** scale model is built to determine the pressure drop across the valve with water as the working fluid. **4-marks**

I. For a certain valve opening, when the velocity of paraffin in the prototype is 3.0 ms^{-1}. What should be the velocity of water in the model for dynamic similarity? **4-marks**

II. What is the pressure drop in the prototype if it is 60 kPa in the model? (The density and the viscosity of paraffin are **800 kgm^{-3}** and **0.002 $kgm^{-1}s^{-1}$** respectively. The kinematic viscosity of water is **1.0×10^{-6} m^2 s^{-1}**).

Q5. **3-marks**

(a). Draw a schematic diagram of boundary layer for a laminar flow over a smooth flat plate clearly showing the boundary layer thickness (δ), velocity profile **u(y)** and the free stream velocity $\mathbf{u_\infty}$. **3-marks**

(b). Explain the significance of the boundary layer displacement thickness (**δ^***) and the momentum thickness (**Θ**), and express them as integrals of the boundary layer velocity profiles for a smooth flat plate. **7-marks**

(c). A laminar boundry layer is given by the following velocity profile,

$$\frac{u}{u_\infty} = \alpha\left(\frac{y}{\delta}\right) + \beta\left(\frac{y}{\delta}\right)^3$$

Where α and β are arbitrary constants

i. Find α and β by using the boundary conditions **7-marks**

ii. Show that (δ^*/δ) =3/8

Q6.

Consider the laminar flow of a fluid layer falling down a plane inclined at an angle θ to the horizontal as shown in Figure Q6.

Starting from Navier-Stokes equations (7-marks)

(a). Show that for steady flow the momentum equation in x-direction can be given expressed as ,

$$\rho g \sin\theta + \mu \frac{\partial^2 u}{\partial y^2} = 0$$

(b). If h is the thickness of the layer in the fully developed stage, show that the velocity distribution is given by,

$$u(y) = \frac{-\rho g \sin\theta}{2\mu}\left(y^2 - h^2\right)$$

7-marks

(c). Show that the volume flow rate per unit width is given by,

$$Q = \left(\frac{\rho g h^3}{3\mu}\right)\sin\theta$$

6-marks

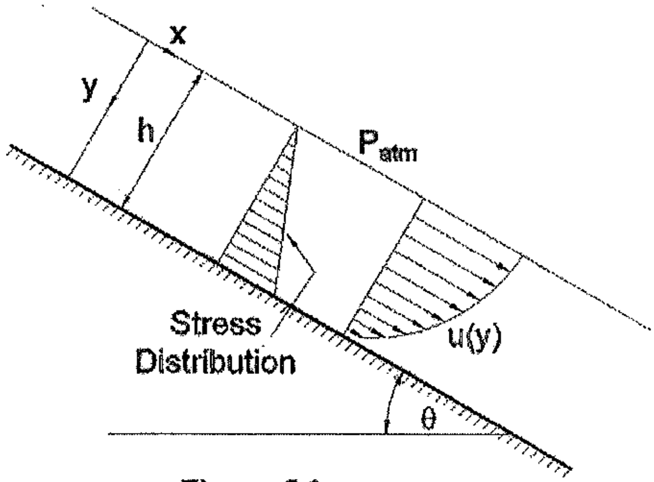

Figure Q6

Q7.

(a). State the applications of potential flow theory. (4-marks)

(b). Define stream function and velocity potential and show that each of them satisfy Laplace's equation.

4-marks

(c). A flow field is characterized by the stream function $\psi = Axy$ where $A = 2s^{-1}$ and the coordinates are measured in meters. Verify that the flow is irrotational and determine the velocity potential ϕ and sketch the streamlines and potential lines.

12-marks

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Q8

(a). Describe the following terms,

I. Doublet

II. Rankine Oval

III. Singularity

6-marks

(b). A source of strength **m** is placed at the origin of a uniform flow field (U) as shown in Figure Q8.

7-marks

Show that the resultant stream function and the velocity potential respectively are given by,

$$\psi = Ur\sin\theta + \frac{m}{2\pi}\theta \quad \text{and} \quad \phi = Ur\cos\theta + \frac{m}{2\pi}\ln(r)$$

(c). A stagnation point is created at (-b,0), show that the equation of the stream line passing through this stagnation point is given by,

$$r = \frac{b(\pi - \theta)}{\sin\theta}$$

7-marks

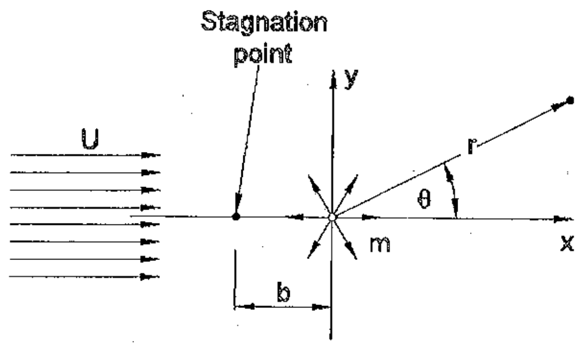

Figure Q8

Nervier-Stokes equations for incompressible flow:-

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \rho g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = \rho g_y - \frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = \rho g_z - \frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

Velocity potential and stream function in cylindrical coordinates:

$$v_r = \frac{\partial\phi}{\partial r} = \frac{1}{r}\frac{\partial\psi}{\partial\theta} \qquad v_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = -\frac{\partial\psi}{\partial r}$$

---------------------- *End of Paper* ----------------------