00078

The Open University of Sri Lanka
Department of Mathematics
Advanced Certificate in Science Programme
MYF2519/MHF2519 - Combined Mathematics I – Level 2

Final Examination 2023/2024

Date: 27-08-2023 **From 9:30 am. To 12:30 pm.**

Answer **All** Questions in **Part A** and Answer **Five** Questions in **Part B.**

PART A

1. (a) Find the domain and range of the function

$$y = \frac{x}{(x+2)(x-1)}, \quad x \neq -2,\ 1\,.$$

(b) Sketch the graph of the above function.

2. The functions $f(x)$ and $g(x)$ are defined by $f: x \to x^2;\ \ x \in \mathbb{R},\ x \geq 0$ and $g: x \to 2x + 5\,;\ x \in \mathbb{R}, x \geq -2.5$.
Find the following:

(a) $f \circ g$,
(b) f^{-1},
(c) g^{-1},
(d) $f^{-1}g^{-1}$.
(e) Solve the equation $f^{-1}g^{-1} = \sqrt{2}$.

3. If one root of the equation $px^2 + qx + r = 0$ is three times the other root show that $3q^2 = 16pr$.

4. Solve the inequality $3(2x + 1) - (x + 3)(x - 1) < 0$.

5. Find the equation of the straight line through the point $(1, -3)$, perpendicular to the line $4x - 3y + 1 = 0$.

6. Find the value of x satisfying the equation

$$\log_a 3 + 2\log_a x - \log_a(x - 1) = \log_a(5x + 2).$$

7. If $(x+3)$ and $(x-1)$ factors of x^3+ax^2+bx-a, find the values of a and b. When a and b takes these values, if the remainder is 15 when this expression is divided by $x-k$, find the value of k.

8. If $p, q \in \mathbb{R}$, and $p \neq q$, prove that the roots of the equation

$x^2-2px+(2p^2-2pq+q^2)=0$ are not real.

9. (a) Given that α and β are two angels such that $\tan\alpha = 2\cot\beta$, show that

$$\tan(\alpha+\beta) = -(\tan\alpha+\tan\beta).$$

(b) Solve the equation $4\tan\theta = 3\sec^2\theta - 7$.

10. In a triangle ABC, $AB = x$, $BC = 4-x$, $AC = x+1$ and $B\hat{A}C = 60^\circ$

(a) Show that $x = \frac{5}{3}$.
(b) Hence, find the area of triangle ABC.

PART B

11. (a) If the polynomial function $f(x) = ax^3 - x^2 - 5x + 3$ is divided by $x+1$, the remainder is 5.
Find the remainder when it is divided by $x-2$. Hence express $F(x) = f(x) - 5$ as a product of linear factors.

(b) Given that $\frac{1}{(px+1)(qx+1)} = \frac{A}{px+1} + \frac{B}{qx+1}$ where p and q are two constants such that

$p \neq q \neq 0$. Without solving for A and B prove that

(i) $A+B=1$ and $Aq+Bp=0$.

(ii) $\frac{1}{(px+1)^2(qx+1)} = \frac{A}{(px+1)^2} + \frac{AB}{(px+1)} + \frac{B^2}{(qx+1)}$.

12. (a) If the roots of the equation $x^2 + bx + c = 0$ are α and β and the roots of the equation $x^2 + \lambda bx + \lambda^2 c = 0$ are γ and δ, show that $(\alpha\gamma + \beta\delta)(\alpha\delta + \beta\gamma) = 2\lambda^2 c(b^2 - 2c)$.

    Find the quadratic equation whose roots are $\alpha\gamma + \beta\delta$ and $\alpha\delta + \beta\gamma$.

    (b) Draw the graph of $y = |x - 1|$ and $y = 3 - |x|$ on same diagram. Hence or otherwise, find the range of values of x satisfying the inequality $|x - 1| > 3 - |x|$.

13. (a) A vertical tower stands on a river bank. From a point on the other bank directly opposite and at a height h above the water level, the angle of elevation of the top of tower is α and the angle of depression of the reflection of the top of tower is β. Assume that water is smooth and the reflection of any object in the water surface will appear to be as far below the surface as the object is above it.

    Prove that the height of the top of the tower above the water is $h\sin(\alpha + \beta)\,\mathrm{cosec}(\beta - \alpha)$ and the width of the river is $2h\cos\alpha\cos\beta\,\mathrm{cosec}\,(\beta - \alpha)$.

    (b) (i) If $t = \tan\frac{\theta}{2}$ then show that

    $\tan\theta = \frac{2t}{1-t^2}$, $\sin\theta = \frac{2t}{1+t^2}$ and $\cos\theta = \frac{1-t^2}{1+t^2}$.

    (ii) If $\tan\theta = \frac{24}{7}$ and θ is acute, calculate $\tan\frac{\theta}{2}$.

14. (a) With the usual notation for a triangle ABC, show that

$$\cos A = \frac{b^2+c^2-a^2}{2bc}.$$

    Prove that

    (i) $b\cos C + c\cos B = a$.

    (ii) for a triangle of area Δ, $ab = 2\Delta\,\mathrm{cosec}\,C$.

    (iii) $a^2 + b^2 = c^2 + 4\Delta\cot C$.

    (iv) Find the remaining sides of a triangle in which one side 5cm the opposite angle is 45°, and area is 15cm^2.

    (b) Show that $\tan^{-1}\left(\frac{1}{n+1}\right) + \tan^{-1}\left(\frac{1}{n^2+n+1}\right) = \tan^{-1}\left(\frac{1}{n}\right)$, where $n \in \mathbb{R}$.

15. (a) (i) Express $f(x) = 5\sin x - 12\cos x$ in the form $f(x) = R\sin(x - \alpha)$ where $R > 0$ and $0 < \alpha < 90^\circ$.

(ii) Find the minimum value $g(x) = \frac{4}{5\sin x - 12\cos x}$.

(iii) Solve the equation $5\sin x - 12\cos x + 3 = 0$.

(b) Solve the following equations

(i) $\sqrt{3}\sin 2x = \cos 2x$

(ii) $\sin 5\theta = \sin 2\theta + \sin\theta$

16. The equations of the altitude (perpendicular line drawn from vertex to the opposite side) AD, BE, and CF of a triangle ABC are $x + y = 0$, $x - 4y = 0$ and $2x - y = 0$ respectively. If the coordinates of A is $(t, -t)$ then show that coordinates of B and C are respectively $\left(-\frac{2t}{3}, -\frac{t}{6}\right)$ and $\left(\frac{t}{2}, t\right)$.

Show also that the locus of the centroid of the triangle ABC is $x + 5y = 0$.

17. Two sides of a parallelogram are given by the equations $y = x - 2$ and $4y = x + 4$. The diagonals of the parallelogram intersect at the origin. Obtain

(i) equations of the remaining sides of the parallelogram.

(ii) the equations of its diagonals.

Also, find the area of the parallelogram.