The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Civil Engineering

Study Programme	: Bachelor of Technology Honours in Engineering
Name of the Examination	: Final Examination
Course Code and Title	**: CVX5443 Structural Analysis**
Academic Year	: 2021/22
Date	: 02nd March 2023
Time	: 09:30-12:30hrs
Duration	: **3 hours**

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **EIGHT (8)** questions in **Five (5)** pages.
3. Answer **ANY FIVE (05)** questions and each question carries **20 Marks.**
4. Answer for each question should commence from a **new page.**
5. This is a **Closed Book Examination.**
6. Answers should be in clear hand writing.
7. Do not use Red colour pen.
8. An electronic non-programmable calculator may be used.
9. Unless otherwise specified, standard nomenclature is used.

QUESTION 1

Stress tensor for a homogenous, isotropic material is given below.

$$\sigma_{ij} = \begin{bmatrix} 10 & -10 & 0 \\ -10 & 10 & 0 \\ 0 & 0 & 20 \end{bmatrix} \text{ in MPa}$$

(i) Graphically represent the above stress tensor. (3 Marks)

(ii) Determine stress invariants (3 Marks)

(iii) Determine principal stresses (4 Marks)

(iv) Obtain volumetric and deviatoric stress tensors (4 Marks)

(v) What should be the minimum uniaxial yield stress values of the material so that it does not fail, according to the **Tresca** and **Von Mises** criteria (6 Marks)

QUESTION 2

(i) Briefly explain why **Airy's stress function** is useful in structural analysis (3 Marks)

(ii) A large thin plate is subjected to certain boundary conditions on its thin edges (with its large faces free of stress), leading to the stress function

$$\emptyset = Ax^3y^2 - Byx^5$$

(a) Use the biharmonic equation to express A in terms of B (6 Marks)
(b) Calculate all stress components (5 Marks)
(c) Calculate all strain components (in terms of B, E, ν) (6 Marks)

QUESTION 3

(i) Describe Mode I, Mode II and Mode III cracks, using clear sketches (4 Marks)

(ii) A single degree of freedom system has a mass of 20 kg and spring constant of 350 N/m. System is given an initial displacement of 10mm and initial velocity is 100mm/s. Find

(a) Natural frequency (4 Marks)

(b) The period of vibration (4 Marks)

(c) The amplitude of vibration (4 Marks)

(d) The time at which third maximum will occur (4 Marks)

QUESTION 4

(i) Briefly explain how **photo-elasticity technique** is used in experimental stress analysis methods. (2 Marks)

(ii) Briefly explain the difference between **metal foil type** and **bonded wire type electrical resistance strain gauges.** (3 marks)

(iii) The stress state of a certain concrete component was determined using a strain rosette as shown in Figure Q4. Due to the loadings, strain gauges gave strain values as $\epsilon_a = -30\ \mu\epsilon$, $\epsilon_b = 45\ \mu\epsilon$, $\epsilon_c = 75\ \mu\epsilon$.

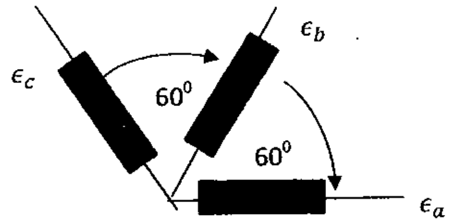

Figure Q4

(a) Draw the Mohr's circle for strains (4 marks)

(b) Determine the in-plane principal strains. (5 marks)

(c) Determine the principal stresses, if the elastic modulus and Poisson's ratio of concrete material are 25GPa and 0.25, respectively. (6 marks)

QUESTION 5

(i) Explain three characteristics of statically indeterminate structures (4 Marks)

(ii) A continuous beam (ABCD) is shown in Figure Q5. Flexural rigidities of members AB and BC are equal to 4EI and member CD is 2EI. Uniformly distributed load (*2W*) is acting on members, AB and BC. There is a concentrated load (*2Wl)* in member CD.

(a) Determine the degree of statical indeterminacy of the beam. (3 Marks)

(b) Draw a released structure. (3 Marks)

(c) Determine the **flexibility matrix** for the drawn released structure. (4 Marks)

(d) Determine bending moment at B. (6 Marks)

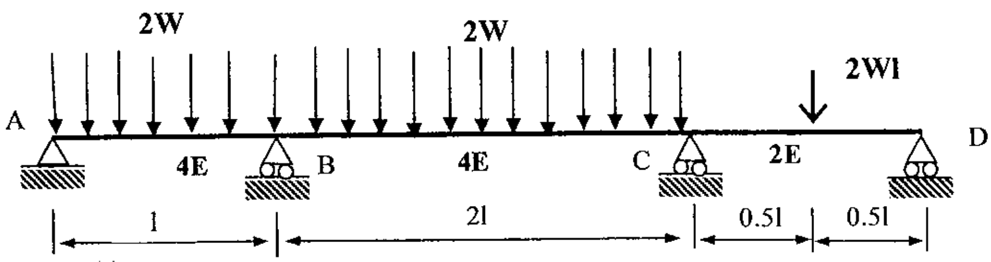

Figure Q5

QUESTION 6

Analyse the continuous beam shown in Figure Q6 using displacement method of structural analysis using following steps

(i)	Find kinematic indeterminacy of the structure	(2 Marks)
(ii)	Draw the structure with independent nodal displacements	(2 Marks)
(iii)	Determine the **stiffness matrix** of the structures.	(4 Marks)
(iv)	Find the free nodal displacements at B using the displacement method.	(6 Marks)
(v)	Using above results, determine the bending moment at B.	(6 Marks)

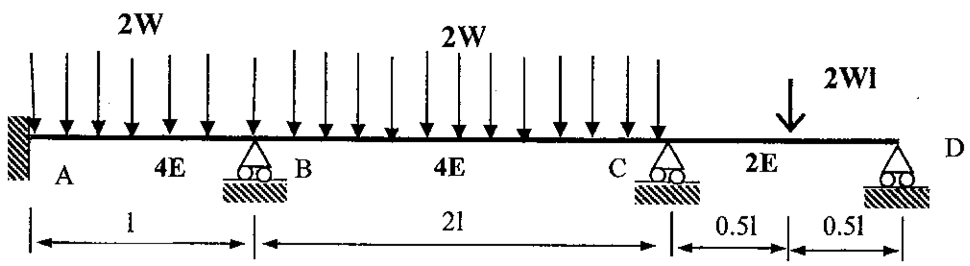

Figure Q6

QUESTION 7

(i) List two assumptions used in theory of thin plates under small deflections (3 Marks)

(ii)A Circular plate of radius "**a**" carries a uniformly distributed load "**q**". The edge at "**r** = **a**", is fixed supported as shown in Figure Q7.

(a) Show that the deflection of the plate is given by $w = \frac{q}{64D}(a^2 - r^2)^2$ (12 Marks)

(b) Find the bending moment at the fixed support (5 Marks)

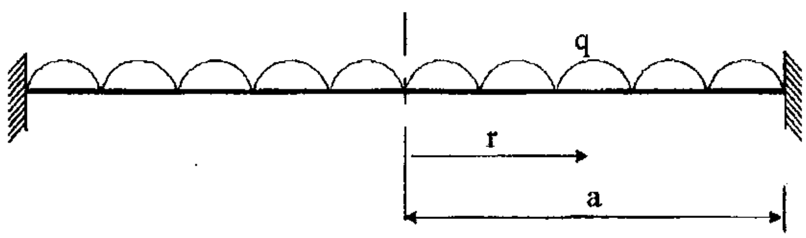

Figure Q7

Hint: Uniformly loaded solid circular plate, with symmetrical boundary conditions is given as

$$\nabla^4 w = \frac{1}{r}\frac{d}{dr}\left\{r\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right]\right\} = \frac{q_0}{D}$$

Where q_0 is a constant.

QUESTION 8

(i) Determine the shape factor for a rectangular cross section (3 Marks)

(ii)A two-storey frame structure is shown in Figure Q8. Dimensions and plastic moments of the columns and beam are given in the figure. Plastic moment (Mp) can be assumed to $3wl^2/4$.

(a) Draw possible locations of plastic hinge formations. (3 Marks)

(b) Determine number of elementary mechanisms (4 Marks)

(c) Draw elementary failure mechanisms. (4 Marks)

(d) Determine load factors for each elementary failure mechanism. (6 Marks)

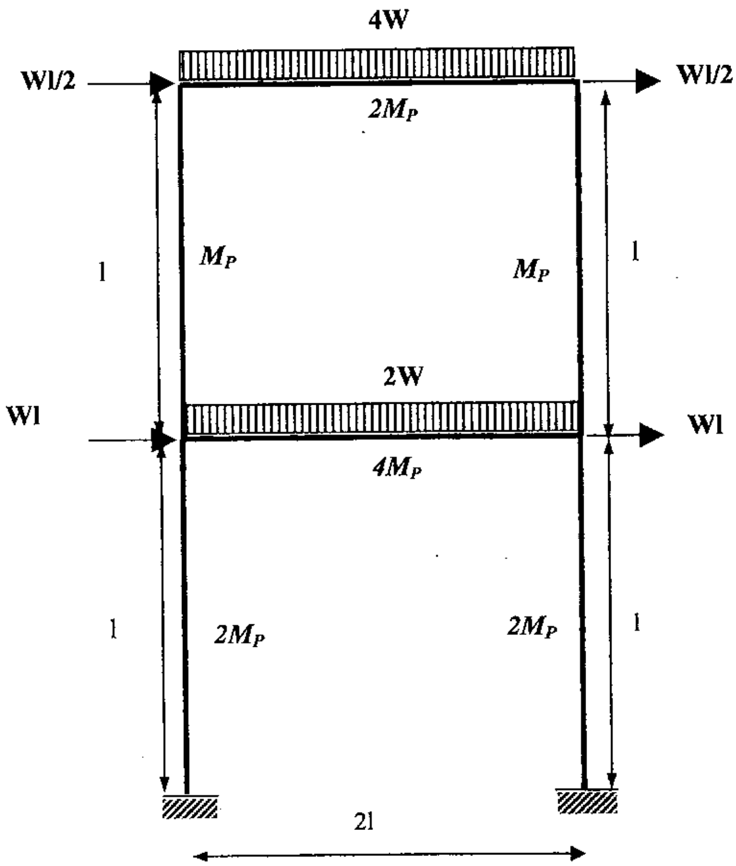

Figure Q8 (Not to scale)

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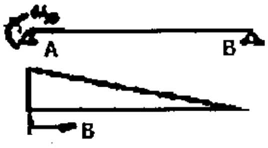
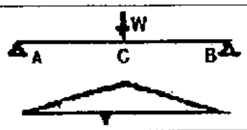
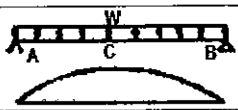
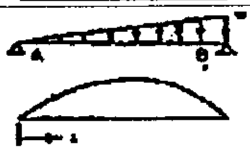
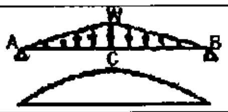
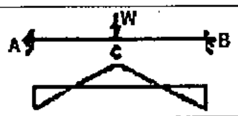
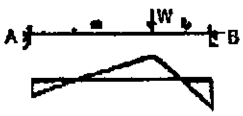
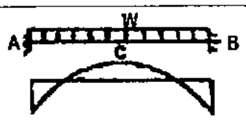
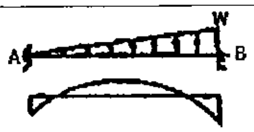
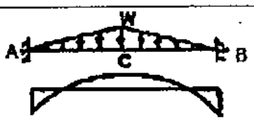
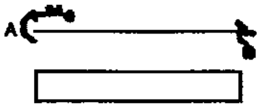
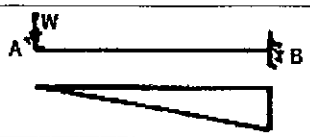
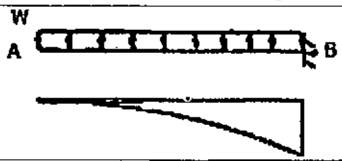
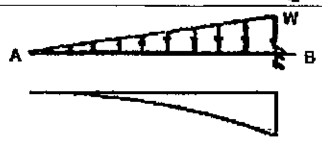
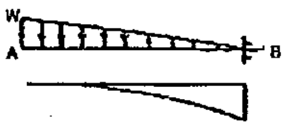
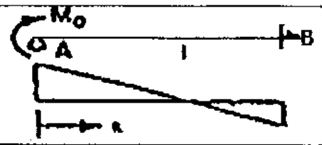
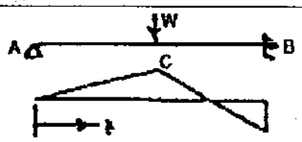
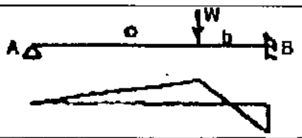
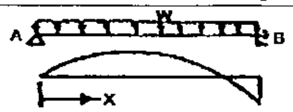
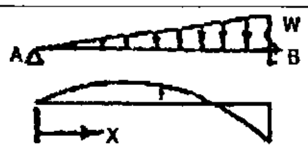
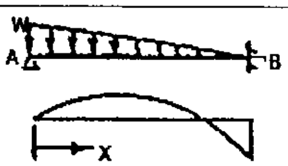
Formulas for Beams				
Structure	Shear ↿⇂	Moment ↺↻	Slope ∢	Deflection ↓
Simply supported Beam				
	$S_A = -\frac{M_o}{L}$	M_o	$\theta_A = \frac{M_o L}{3EI}$ $\theta_B = \frac{M_o L}{6EI}$	$Y_{max} = 0.062\frac{M_o L^2}{3EI}$ $Y_c = \frac{WL^3}{48EI}$
	$S_A = \frac{W}{2}$	$M_o = \frac{WL}{4}$	$\theta_A = -\theta_B = \frac{WL^2}{16EI}$	$Y_c = \frac{WL^3}{48EI}$
	$S_A = \frac{Wb}{L}$ $S_B = \frac{Wn}{L}$	$M_o = \frac{Wab}{L}$	$\theta_A = \frac{Wab}{6EIL}(L+b)$ $\theta_B = -\frac{Wab}{6EIL}(L+a)$	$Y_o = \frac{Wa^2b^2}{3EIL}$
	$S_A = \frac{WL}{2}$	$M_c = \frac{WL^2}{8}$	$\theta_A = -\theta_B = \frac{WL^3}{24EI}$	$Y_c = \frac{5WL^4}{384EI}$
	$S_A = \frac{WL}{6}$ $S_B = \frac{WL}{3}$	$M_{max} = 0.064Wl^2$ at $x = 0.577L$	$\theta_A = \frac{7WL^3}{360EI}$ $\theta_B = -\frac{8WL^3}{360EI}$	$Y_{max} = 0.00652\frac{WL^4}{EI}$ at $x = 0.519L$
	$S_A = \frac{WL}{4}$	$M_c = \frac{WL^2}{12}$	$\theta_A = -\theta_B = \frac{5W\ L^3}{192EI}$	$Y_c = \frac{WL^4}{120EI}$
Fixed Beams				
	$S_A = \frac{W}{2}$	$M_c = \frac{WL}{8}$	$\theta_A = \theta_B = 0$	$Y_c = \frac{WL^3}{192EI}$
	$S_A = \frac{Wb^2}{L^3}(3a+b)$ $S_B = \frac{Wa^2}{L^3}(3b+a)$	$M_A = -\frac{Wab^2}{L^2}$ $M_B = -\frac{Wba^2}{L^2}$	$\theta_A = \theta_B = 0$	$Y_o = \frac{Wa^3b^3}{3EIL^3}$
	$S_A = \frac{WL}{2}$	$M_A = M_B = -\frac{WL^2}{12}$	$\theta_A = \theta_B = 0$	$Y_c = \frac{WL^4}{384EI}$
	$S_A = \frac{3WL}{20}$ $S_B = \frac{7WL}{20}$	$M_A = -\frac{WL^2}{30}$	$\theta_A = \theta_B = 0$	$Y_{max} = 0.00131\frac{WL^4}{EI}$ at $x = 0.525L$
	$S_A = \frac{WL}{4}$	$M_A = M_B = -\frac{5WL^2}{96}$	$\theta_A = \theta_B = 0$	$Y_c = \frac{0.7WL^4}{384EI}$

Structure	Shear ↿⇂	Moment ↻↺	Slope	Deflection ↓
Cantilever Beam				
	0	M_o	$\theta_A = \frac{M_o L}{EI}$	$Y_A = \frac{M_o L^2}{2EI}$
	W	$M_B = -WL$	$\theta_A = -\frac{WL^2}{2EI}$	$Y_A = \frac{WL^3}{3EI}$
	$S_B = -WL$	$M_B = -\frac{WL^2}{2}$	$\theta_A = \frac{WL^3}{6EI}$	$Y_A = \frac{WL^4}{8EI}$
	$S_B = -\frac{WL}{2}$	$M_B = -\frac{WL^2}{6}$	$\theta_A = -\frac{WL^3}{24EI}$	$Y_A = \frac{WL^4}{8EI}$
	$S_B = -\frac{WL}{2}$	$M_B = -\frac{WL^2}{2}$	$\theta_A = -\frac{WL^3}{8EI}$	$Y_A = \frac{11WL^4}{120EI}$
Propped Cantilever				
	$S_A = \frac{3M_0}{2L}$	$M_B = \frac{M_0}{2}$	$\theta_A = -\frac{M_0 L}{4EI}$	$Y_{max} = \frac{W_0 L^2}{27EI}$ at $x = \frac{L}{3}$
	$S_A = -\frac{5W}{16}$	$M_B = -\frac{3WL}{16}$ $M_c = -\frac{5WL}{32}$	$\theta_A = -\frac{WL^2}{32EI}$	$Y_{max} = 0.00932\frac{WL^3}{EI}$ at $x = 0.447L$
	$S_A = \frac{Wb^2}{2L^3}(a + 2L)$ $S_B = \frac{Wa}{2L^3}(3L^2 - a^2)$	$M_B = -\frac{Wab}{L^2}\left(a + \frac{b}{2}\right)$	$\theta_A = -\frac{Wab^3}{4EIL}$	$Y_0 = \frac{Wa^2b^3}{12EIL^3}(3L + a)$
	$S_A = \frac{3WL}{8}$	$M_B = -\frac{WL^2}{8}$	$\theta_A = -\frac{WL^3}{48EI}$	$Y_{max} = 0.0054\frac{WL^4}{EI}$ at $x = 0.422L$
	$S_A = \frac{WL}{10}$	$M_{max} = 0.03WL^2$ at $x = 0.447L$ $M_B = -\frac{WL^2}{15}$	$\theta_A = -\frac{WL^3}{120EI}$	$Y_{max} = 0.00239\frac{WL^4}{EI}$ at $x = 0.447L$
	$S_A = \frac{11WL}{40}$	$M_{max} = 0.0423WL^2$ at $x = 0.329L$ $M_B = -\frac{7WL^2}{120}$	$\theta_A = -\frac{WL^3}{80EI}$	$Y_{max} = 0.00305\frac{WL^4}{EI}$ at $x = 0.402L$