The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Civil Engineering

Study Programme	: Bachelor of Technology Honours in Engineering
Name of the Examination	: Final Examination
Course Code and Title	**: CVX 5531 Mechanics of Fluids**
Academic Year	: 2021/22
Date	: 11th January 2022
Time	: 13:30-16:30hrs
Duration	: **03 hours**

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **EIGHT (08)** questions on **FIVE (05)** pages.
3. Answer any **FIVE (05)** questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Necessary additional information is provided.
6. This is a Closed Book Test (CBT).
7. Answers should be in clear hand writing.
8. Do not use Red colour pen.
9. Take,

   Density of water = 1000 kgm^{-3} Acceleration due to gravity = 9.81 ms^{-2}

   Kinematic viscosity of water = 8.36 x 10^{-05} m^2/s at 28 ^oC

Question 01

(a) An unsteady incompressible flow field is given by, $u = t^2 + 3y$ and $v = 4t + 5x$. Determine the acceleration at the point (4,3) at time $t = 3$.

(06 marks)

(b) Confirm that the following flow exists and obtain the associated stream function.

$u = -cx/y$

$v = c \ln xy$ (08 marks)

(c) A velocity potential for a 2-Dimensional flow is given by $\phi = (x^2 - y^2) + 3xy$. Compute the flow rate between the streamlines passing through points (1,1) and (2,3).

(06 marks)

Question 02

(a) The following velocity profile is approximated for the boundary layer in a flow over a flat plate,

$$\frac{u}{U_S} = 2\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^3 + \left(\frac{y}{\delta}\right)^4$$

where, U_S is the free stream velocity. δ is the boundary layer thickness and y is the distance to a point in the boundary from the flat plate in the normal direction to the flow. The shear stress on the plate is given by,

$$\tau_0 = \frac{37}{315}\rho U_S^2 \frac{\partial \delta}{\partial x}.$$

Show that the friction drag, F_D on one side of the plate is given by,

$$F_D = 0.6855 b \mu U_S \sqrt{\rho U_S L/\mu}$$

where, ρ and μ are the density and the dynamic viscosity of the flowing fluid, respectively. b and L are the width and the length of the flat plate, respectively.

(10 marks)

(b) A kite is in the form of a rectangular airfoil with a chord length of 75 cm and a width of 50 cm. It is maintained at an angle of 12^0 to the horizontal and the string makes an angle 45^0 to the vertical. The weight of the kite is 0.1 kg. If the wind speed is 12 km/hr and the drag coefficient, C_D is 0.23, estimate,

(i) The tension in the string

(ii) The lift coefficient, C_L, if the density of air is 1.2 kg/m^3

(10 marks)

Question 03

(a) The operating variables of a pump are the flow rate Q, the head provided H, the rotational speed of the impeller N, the impeller diameter D, the gravitational acceleration g, the density ρ and the dynamic viscosity μ of the fluid. Obtain the following relationship for the non-dimensional quantities associated using Buckingham's Pi-theorem.

$$\varphi\left\{\frac{Q}{ND^3}, \frac{g}{N^2D}, \frac{\mu}{ND^2\rho}, \frac{H}{D}\right\} = 0$$

(12 marks)

(b) A centrifugal pump with specific speed, 20 running at 1150 rpm delivers 3.6 m^3/s. The impeller diameter is 0.3 m. Determine the flow rate, head, and the specific speed if the pump runs at 1750 rpm.

(08 marks)

Question 04

A Pelton wheel turbine is working under a gross head of 250 m. The water is supplied through a penstock of diameter 0.9 m and length 3 km from reservoir to the Pelton wheel. The coefficient of friction, f for the penstock is 0.005. The jet of water of diameter 120 mm strikes the buckets of the wheel and gets deflected through an angle 165^0. The relative velocity of water at outlet is reduced by 10% due to inside roughness of the buckets. If the velocity of the buckets is 0.45 times the jet velocity at inlet and mechanical efficiency is 83%, determine,

(a) power given to the runner,

(15 marks)

(b) shaft power,

(02 marks)

(c) hydraulic efficiency.

(03 marks)

Additional Information:

The power given to the runner of a turbine is given by, $\rho Q(v_{w1} - v_{w2})u$.

Question 05

(a) Show that the velocity components, u_r and u_θ corresponding to the flow around a circular cylinder of radius a with a circulation, Γ are given by,

$$u_r = U\left(1-\frac{a^2}{r^2}\right)\cos\theta$$

$$u_\theta = -\left[U\left(1+\frac{a^2}{r^2}\right)\sin\theta+\frac{\Gamma}{2\pi r}\right]$$

where, U is the free stream velocity.

When the complex potential for a 2D potential flow having no rigid boundaries in the z-plane is given by, $f(z)$, the complex potential for a flow around a circular cylinder whose radius, $r=a$ is given by,

$$F(z) = f(z) + \overline{f\left(\frac{a^2}{z}\right)}$$

(13 marks)

(b) If the stream function associated with a *doublet* in polar coordinates is given by,

$$\psi = -K\frac{\sin\theta}{r}$$

Where K is the strength of the *doublet*. Determine,

(i) The velocity and

(ii) The value of the stream function

for a point P (0.6,1.2) is situated in the flow field of a *doublet* of strength 6 m^3/s.

(07 marks)

Question 06

(a) A semicircular duct having a radius, R carries water with a free surface inside as shown in Figure Q6. Show that the maximum discharge in this duct is observed when depth of flow, h equals $0.927R$ [use Manning's formula].

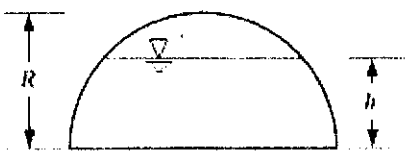

Figure Q6

(09 marks)

(b) A *wide rectangular* channel carries a water flow of 3.50 m^3/s per metre width. If the flow is uniform upstream and channel slope is 1:2000;

(i) Calculate the normal depth of flow upstream. Take Manning's roughness coefficient as 0.015.

(ii) Assuming flow approaches at normal depth, calculate the minimum rise in the floor at a downstream section required to produce critical flow conditions.

(iii) If the actual rise in floor is 0.20 m, determine the change in depth of flow due to such rise. Justify your answer with reference to specific energy vs. depth of flow relationship.

(11 marks)

Additional Information:

The Manning's equation for uniform flow is given by: $v = \frac{1}{n} R^{2/3} S_0^{1/2}$.

Question 07

(a) Explain the phenomenon of cavitation with reference to pumps. In addition, briefly discuss the precautions, which can be taken against it.

(05 marks)

(b) A centrifugal pump with an outer impeller diameter 0.6 m runs at 250 r.p.m and pumps 0.8 m^3/s, the average lift being 2 m. The angle which the vanes make at exit with the impeller is 28^0 and the radial velocity of flow 1.2 m/s. The inner diameter of the impeller is 0.3 m. The power supplied to the pump is 40 kW and the mechanical efficiency is estimated to be 93%.

(i) Determine the manometric efficiency and the overall efficiency.

(10 marks)

(ii) The flow is now reduced by 50% using a regulator. Estimate the percentage change in manometric efficiency, if the average lift is increased to 2.4 m.

(03 marks)

(iii) Determine the least pump speed required to start pumping against a head of 2 m.

(02 marks)

Additional Information:

The theoretical head provided by a pump is given by, $\frac{u_2 v_{w2}}{g}$.

Question 08

A reservoir supplies water at a steady mean velocity, v to the turbine of a power plant through a long pipe in which the friction loss may be assumed to be proportional to v^2. The system is protected against high-pressure transients by means of a surge tank.

(a) If the flow to the turbine is stopped instantaneously, show that at any time the level in the surge tank, z is related to by an equation of the form,

$$2C_1C_2\left[v\frac{d^2v}{dz^2}+\left(\frac{dv}{dz}\right)^2\right]-2C_1v\frac{dv}{dz}+1=0$$

The above differential equation has the following solution

$v^2=\frac{1}{C_1}(z+C_2)+C_3e^{\frac{z}{C_2}}$

where, C_1, C_2 and C_3 are constants.

(12 marks)

(b) Water from a reservoir is supplied to a power plant through a pipe of diameter 0.75 m and length 1500 m at a steady flow rate of 1.2 m^3/s. A surge tank of diameter 3 m is connected 100 m upstream of the turbine. If the base of the surge tank is 20 m below the free surface of the reservoir, estimate the height of tank required to accommodate instantaneous complete shut-down of the system without overflowing. The friction factor may be assumed constant and equal to 0.006.

(08 marks)

Friction head loss along a pipe, $h_f=4f\frac{l}{d}\frac{v^2}{2g}$