063

The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Mechanical Engineering

Study Programme	Bachelor of Technology Honours in Engineering
Name of the Examination	Final Examination
Course Code and Title	**DMX5201 Advance Engineering Mechanics**
Academic Year	2021/22
Date	02^{nd} February 2023 (Thursday)
Time	09.30 - 12.30 hrs
Duration	**03 hours**

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **Five (5)** Pages.
3. Answer **All Questions** in **SECTION A** and any **Three (3)** questions from **SECTION B**
4. This is a Closed Book Test (CBT).
5. Answers should be in clear handwriting.
6. Do not use Red color pen.

SECTION A

QUESTION A1 (20 marks)

Figure QA1 (a) shows a simple idealization of a force sensor, which is used to measure the force **F**, by providing an electrical signal. The electrical signal is proportional to the length '*s*' of the spring.

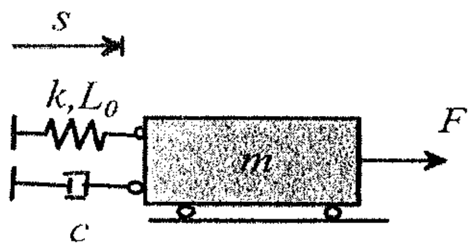

Figure QA1 (a)

At time **t** = **0** the system is at rest, and the applied force **F** = **0**. When time **t** = **1** s an impulsive force of F = **100N** is applied to the mass **m** and the motion of the system is observed. The Figure QA1 (b) shows the variation of spring length **s** with time for **0** < **t** < **5** s.

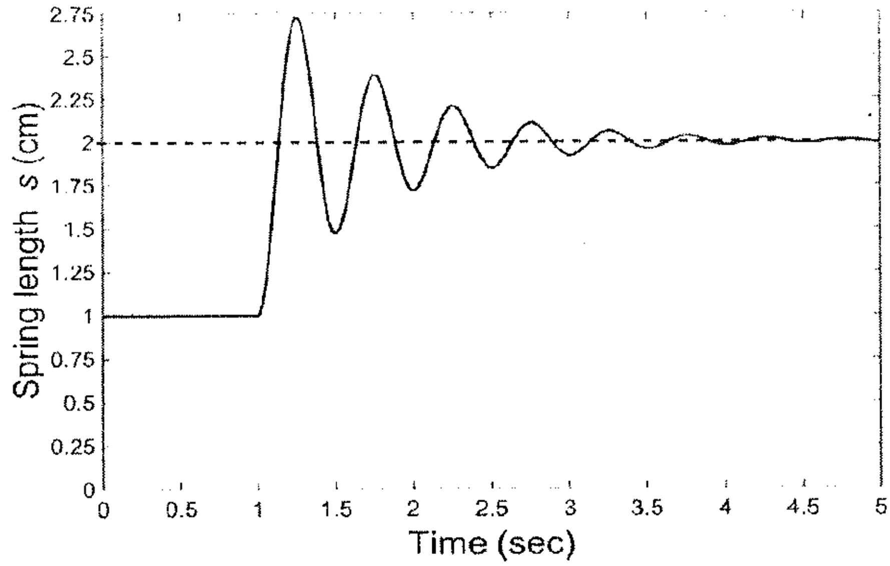

Figure QA1 (b)

(a) Using the graph provided (Figure QA1 (b)), answers the following questions.

i. What is the period (**T**) of vibration?
ii. What is the damped natural frequency (**ω_d**) of the systems?
iii. Find the log decrement of the vibration (be careful to use the correct origin) – **δ.**
iv. Calculate the damping factor of the system – **ζ.**
v. Find the undamped natural frequency of the system - **ω_n**.
vi. Find the un-stretched length of the spring - **L_0**.
vii. Calculate the spring stiffness – **k.**
viii. What is the value of the mass – **m?**
ix. Calculate the dashpot coefficient – **c.**

(b) The sensor is now used to measure a force that vibrates harmonically **$F(t) = F_0 \sin(\omega t)$**. The Figure QA1 (c) shows the steady-state variation of the spring length **s** with time. Calculate the amplitude of the force.

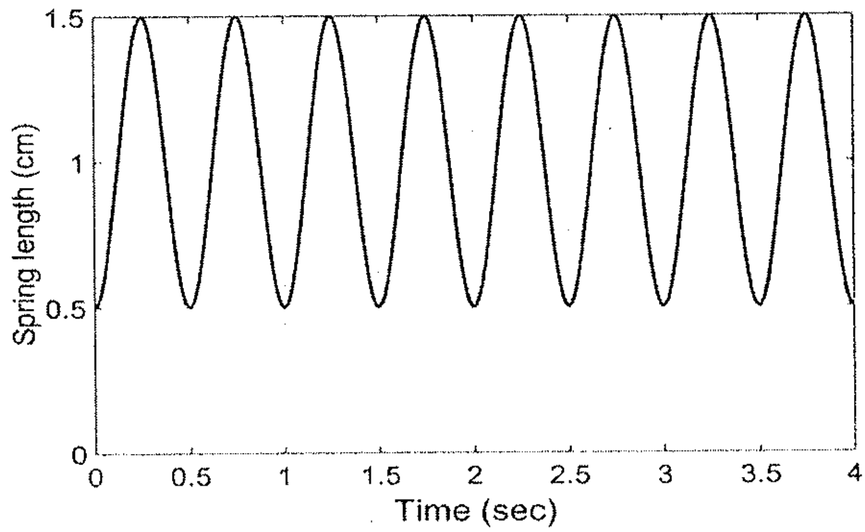

Figure QA1 (c)

Note: The frequency of the force is equal to the natural frequency (the period of vibration is equal to the period in the Figure QA1(b)).

QUESTION A2 (20 marks)

Determine the moment of inertia of the bent rod shown in Figure QA2 about the **A-a** axis. The mass of each of the three segments is given in the figure.

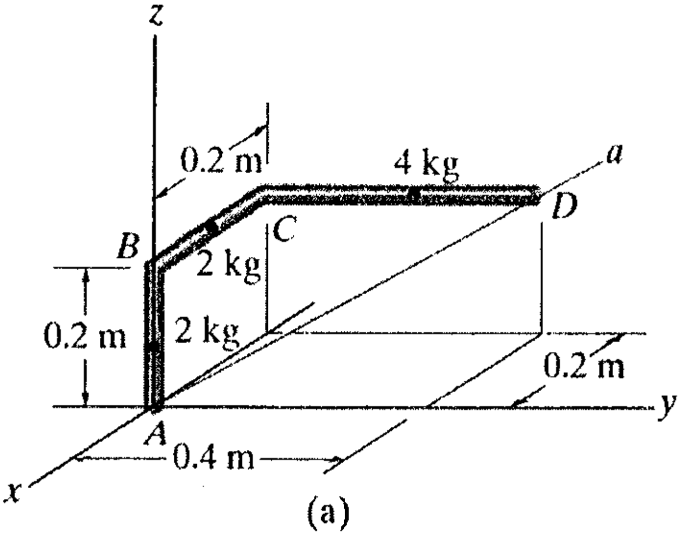

Figure QA2

SECTION B

QUESTION B1 (20 marks)

Figure QB1 shows a vibrating system consist of two masses m_1 and m_2. This represent a simple multi degree of freedom vibrating system. Such systems can be analyzed using Newton's Laws or using Lagrange Formulation. Lagrange formulation is sometimes called as the energy method.

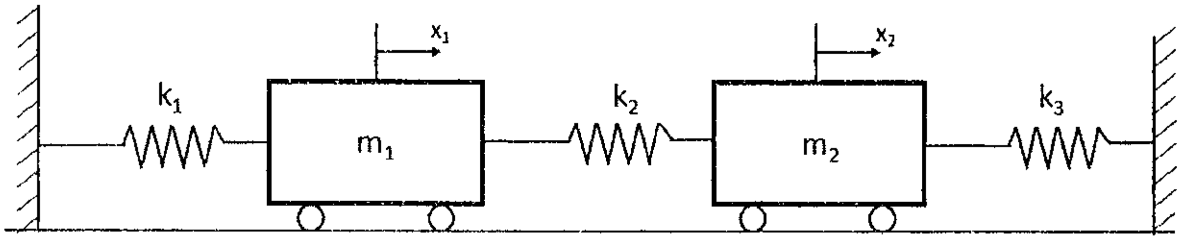

Figure QB1

(a) Write the equations for total potential energy (**U**) and for total kinetic energy (**T**) of the system given in Figure QB1.

(b) Derive the governing equation of motion for the system in matrix form using Lagrange formulation. For a conservative system it is given that,

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\right) - \left(\frac{\partial T}{\partial x_i}\right) + \left(\frac{\partial U}{\partial x_i}\right) = 0$$

(c) If $\mathbf{m_1} = \mathbf{10}$ kg, $\mathbf{m_2} = \mathbf{20}$ kg, $\mathbf{k_1} = \mathbf{k_2} = \mathbf{1x10^5}$ N/m and $\mathbf{k_3} = \mathbf{2x10^5}$ N/m, find the natural frequency ω_n and mode shapes of the system.

(d) When the motion starts (at t = 0) $x_1 = \mathbf{0.001}$ m, $x_2 = \mathbf{0}$, $\dot{x}_1 = \mathbf{0}$ and $\dot{x}_2 = \mathbf{0}$. Find the complete solution of this vibrating system. The general solution of such system with usual notations can be written as

$$\boldsymbol{x}(t) = \ A_1 x_1 \sin(\omega_1 t + \varphi_1) + A_2 x_2 \sin\,(\omega_2 t + \varphi_2)$$

QUESTION B2 (20 marks)

A cantilever of length **L** and mass **m** per unit length has a vertical prop at the free end so that the fixed end and the free end of the cantilever are at the same level. The static deflection y of the propped cantilever at a distance x from the fixed end is given by,

$$y = \frac{mg}{48EI}(2x^4 + 3L^2x^2 - 5Lx^3)$$

where **EI** is the flexural rigidity of the beam. If the cantilever takes the same form as the static deflection curve when it vibrates transversely, determine the natural frequency of transverse vibration of the propped cantilever in terms of **EI, L** and **m**.

QUESTION B3 (20 marks)

A torsional vibration system consists of four rotors, 1, 2, 3, and 4, mounted on a steel shaft of diameter 50 mm. Moments of inertia of the rotors and the lengths of the sections of the shaft are given in the Table QB3(a) and QB3(b).

Moments of Inertia of Rotors ($\mathbf{kgm^2}$)	
Rotor 1	0.60
Rotor 2	0.20
Rotor 3	0.10
Rotor 4	0.25

Table QB3(a)

Lengths of Sections of Shafts between Rotors (m)	
Between rotors 1 and 2	0.50
Between rotors 2 and 3	0.25
Between rotors 3 and 4	1.00

Table QB3(b)

To find the natural frequencies and the mode shapes, it is decided to use Holzer method and the first natural frequency of the system is suggested as **62.5** Hz. The modulus of rigidity of the shaft material is given as **80** GN/m^2

(a) Find the approximate value of frequency and determine whether it is high or low.

(b) Sketch the mode of vibration at this frequency and find the position of the node.

QUESTION B4 (20 marks)

(a) Explain, what a Vibration Exciter is? Name three different types of vibration exciters.

(b) With the aid of sketches, briefly describe one of the vibration exciters you mentioned above.

(c) Explain, what is machine condition monitoring stating its advantages and disadvantages?

(d) Name four different display formats and analysis tools used in vibration signal analysis. Briefly explain one of them.

QUESTION B5 (20 marks)

A motor and attached rod **AB** have the angular motions shown in Figure QB5. A collar **C** on the rod is located **0.25** m from **A** and is moving downward along the rod with a velocity of **3** m/s and an acceleration of **2** m/s^2. At the given instance the motor shaft rotates with an angular velocity $\omega_M = 3$ rad/s and angular acceleration of $\dot{\omega}_M = 1$ rad/s^2. The circular platform which the motor is attached rotates around its center axis with an angular velocity $\omega_p = 5$ rad/s and angular acceleration of $\dot{\omega}_p = 2$ rad/s^2. Determine the velocity and acceleration of the collar **C** relative to the fixed coordinates system at this instant.

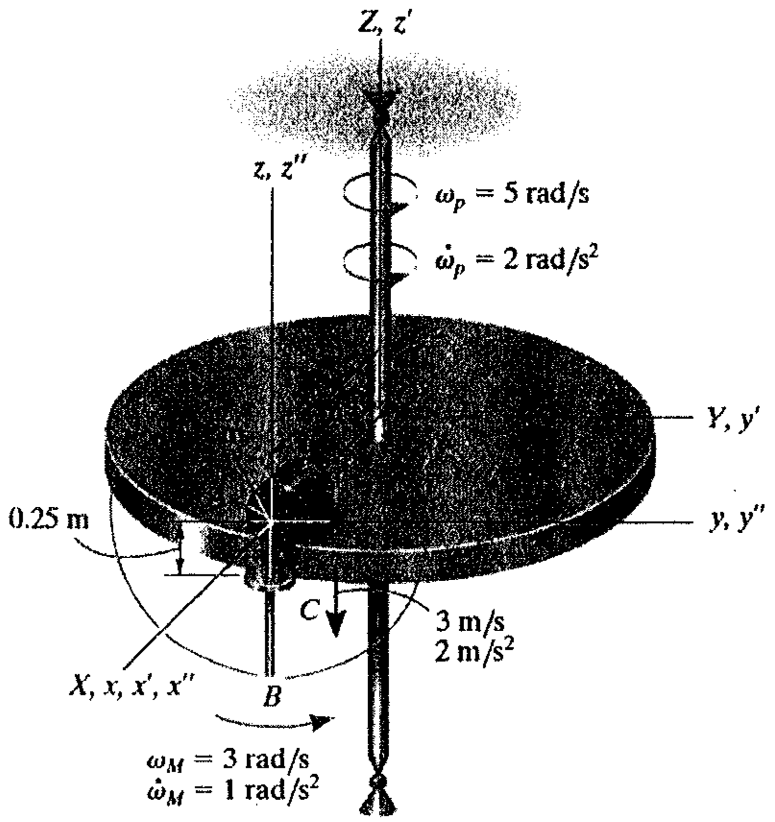

Figure. QB5