The Open University of Sri Lanka
Department of Electrical and Computer Engineering
ECX6242 – Modern Control Systems
Final Examination – 2016/2017
Bachelor of Technology Honours in Engineering

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Answer **five** questions by selecting **at least two questions** from each of the **sections A** and **B.**

Section A

Q1.

(a) Describe what are the advantages of state space modelling?

(b) Consider the system represented by the equation

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -10 & -7 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [1 \quad 0] x(t)$$

Find the transfer function of the system.

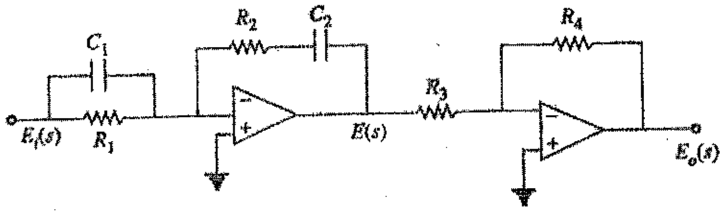
(c) Check for the controllability of the system.

Q2.

(a) Explain the structure of a PID controller.

(b) Briefly describe a PID controller tuning methodology.

(c) Show that the following circuit is a PID controller.



Q3.

(a) Briefly describe Lyapunov's direct method for the determination of the stability of non-linear systems.

(b) Consider the scalar system $\dot{x} = ax^3$

(i) Show that Lyapunov's linearization method fails to determine the stability of the origin.

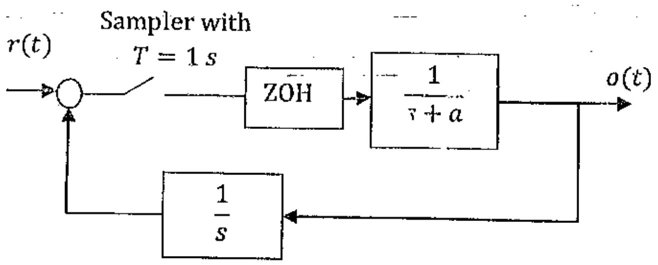
(ii) Use Lyapunov's function $V(x) = x^4$ to show that the system is stable for $a < 0$ and unstable for $a > 0$.

(iii) What can you say about the system stability for $a = 0$?

Q4.

(a) Explain Sample and Hold (SOH) as applied to discrete systems.

(b) Determine the output in discrete form when a unit step is applied to the input of the following closed-loop system.



Section B

The questions in this section are based on the research paper reproduced at the end of this question paper. Devote at least half an hour to reading through the paper. Use your own words in your answers so as to demonstrate that you have understood the concepts described in the paper, do not copy extracts from the paper itself.

Q5. Explain the structures of PI, PD and PID controllers and compare their applications.

Q6. Briefly explain the proposed tuning methodology in this paper.

Q7. Find the tuning parameters of the PID controller for $G(s) = \frac{2e^{-0.3s}}{s+1}$ using Ziegler-Nichols and proposed ISE method. Compare the results.

Q8. Discuss the advantages and disadvantages of the proposed methods over Ziegler-Nichols and Cohen-Coon methods.

Laplace transform	Corresponding z-transform
$\frac{1}{s}$	$\frac{z}{z-1}$
$\frac{1}{s^2}$	$\frac{Tz}{(z-1)^2}$
$\frac{1}{s^3}$	$\frac{T^2 z(z+1)}{2(z-1)^3}$
$\frac{1}{s+a}$	$\frac{z}{z-e^{-aT}}$
$\frac{1}{(s+a)^2}$	$\frac{Tze^{-aT}}{(z-e^{-aT})^2}$
$\frac{a}{s(s+a)}$	$\frac{z(1-e^{-aT})}{(z-1)(z-e^{-aT})}$
$\frac{b-a}{(s+a)(s+b)}$	$\frac{z(e^{-aT}-e^{-bT})}{(z-e^{-aT})(z-e^{-bT})}$
$\frac{(b-a)s}{(s+a)(s+b)}$	$\frac{(b-a)z^2-(be^{-aT}-ae^{-bT})z}{(z-e^{-aT})(z-e^{-bT})}$
$\frac{a}{s^2+a^2}$	$\frac{z \sin aT}{z^2-2z\cos aT+1}$
$\frac{s}{s^2+a^2}$	$\frac{z^2-z\cos aT}{z^2-2z\cos aT+1}$
$\frac{s}{(s+a)^2}$	$\frac{z[z-e^{-aT}(1+aT)]}{(z-e^{-aT})^2}$

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OPTIMAL TUNING OF PID CONTROLLERS FOR FIRST ORDER PLUS TIME DELAY MODELS USING DIMENSIONAL ANALYSIS

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Abstract: Using dimensional analysis and numerical optimisation techniques, an optimal method for tuning PID controllers for first order plus time delay systems is presented. Considering integral square error (ISE), integral absolute error (IAE) and integral time absolute error (ITAE) performance criteria, optimal equations for obtaining PID parameters are proposed. Simulation results show that the proposed method has a considerable superiority over conventional techniques. In addition, the closed loop system shows a robust performance in the face of model parameters uncertainty.

Keywords: PID controller, FOPTD model, dimensional analysis, Ziegler-Nichols method, Cohen-Coon method, optimisation, robustness.

1. INTRODUCTION

It is generally believed that PID controllers are the most popular controllers used in process control. Because of their remarkable effectiveness and simplicity of implementation, these controllers are overwhelmingly used in industrial applications [1], and more than 90% of existing control loops involve PID controllers [2]. Since the 1940s, many methods have been proposed for tuning these controllers, but every method has brought about some disadvantages or limitations [1]. As a result, the design of PID controllers still remains a challenge before researchers and engineers.

A PID controller has the following transfer function:

$$K(s) = K_c\left(1+\frac{1}{T_i s}+T_d s\right) \qquad (1)$$

Obviously, this transfer function is improper and cannot be used in practice, because its gain is increased with no bound as frequency increases. Practical PID controllers limit this high frequency gain using a first order low pass filter. Therefore, a practical PID controller has the following transfer function:

$$K(s) = K_c\left(1+\frac{1}{T_i s}+\frac{T_d s}{\gamma s+1}\right) \qquad (2)$$

where γ is a small number and may be set as 10% of the value of the derivative term [3]. The aim of PID control design is to determine PID parameters (K_c, T_i and T_d) to meet a given set of closed loop system performance requirements.

2. FIRST ORDER PLUS TIME DELAY MODELS

A large number of industrial plants can approximately be modelled by a first order plus time delay (FOPTD) transfer function as follows:

$$G(s) = \frac{Ke^{-\tau_d s}}{Ts+1} \qquad (3)$$

To design PID controllers for this important category of industrial plants, various methods have been suggested during the past sixty years. Ziegler-Nichols and Cohen-Coon design methods are the most prominent techniques mentioned in most control textbooks.

3. CONVENTIONAL DESIGN TECHNIQUES

3.1. ZIEGLER-NICHOLS METHODS

The Ziegler-Nichols design methods are the most popular methods used in process control to determine the parameters of a PID controller. Although these methods were presented in the 1940s, they are still widely used.

The first method of Ziegler and Nichols known as the *continuous cycling method* was proposed in 1942 [4]. In this method, integration and derivative terms of the controller are disabled and the proportional gain is increased until a continuous oscillation occurs at gain K_u for the closed loop system. Considering K_u and its related oscillating period, T_u, the PID parameters can be calculated from the following equation:

$$\begin{aligned} K_c &= 0.6K_u \\ T_i &= 0.5T_u \\ T_d &= 0.125T_u \end{aligned} \qquad (4)$$

A clear deficiency of this method is that it does not work for plants whose root loci do not cross the imaginary axis for any value of gain.

In 1943, the second method of Zeigler and Nichols known as the *process reaction curve method* was proposed to determine the PID parameters for an FOPTD model [5]. In this method, the PID parameters are calculated as:

$$K_c = \frac{1.2T}{K\tau_d}$$
$$T_i = 2\tau_d \qquad (5)$$
$$T_d = 0.5\tau_d$$

A common disadvantage of the Ziegler-Nichols methods is that the resulting closed loop system is often more oscillatory than desirable [6].

3.2. COHEN-COON METHOD

In order to provide closed loop responses with a damping ratio of 25%, Cohen and Coon [7] suggested the design equation (6) for an FOPTD model. Similar to the Ziegler and Nichols methods, this technique sometimes brings about oscillatory responses.

$$K_c = \frac{\frac{\tau_d}{4T} + \frac{4}{3}}{K\frac{\tau_d}{T}}$$
$$T_i = \tau_d \frac{\frac{3\tau_d}{4T} + 4}{\frac{\tau_d}{T} + \frac{13}{8}} \qquad (6)$$
$$T_d = \tau_d \frac{2}{\frac{\tau_d}{T} + \frac{11}{2}}$$

4. PROPOSED METHOD

The aim of this paper is to propose a set of formulas for tuning a PID controller for an FOPTD model. Therefore, as shown in equation (7), the PID parameters should be defined based on the model parameters:

$$K_c = f_1(K, \tau_d, T)$$
$$T_i = f_2(K, \tau_d, T) \qquad (7)$$
$$T_d = f_3(K, \tau_d, T)$$

The problem is that it is quite difficult to determine these functions. Therefore, it was proposed to use *dimensional analysis* to reduce the number of parameters. Dimensional analysis is a mathematical tool often applied in physics and engineering to simplify a problem by reducing the number of variables to the smallest number of essential parameters [8].

Definition 1:
A dimensionless number is a pure number without any physical unit. Such a number is typically defined as a product or ratio of quantities that have units, in such a way that all units can be cancelled.

Theorem 1 (Buckingham pi-theorem):
Any physically meaningful equation such as

$$\alpha(R_1, R_2, ..., R_n) = 0 \qquad (8)$$

with $R_j \neq 0$ ($j = 0,1,...,n$) is equivalent to an equation of the form

$$\beta(\pi_1, \pi_2, ..., \pi_k) = 0 \qquad (9)$$

where π_i ($i = 0,1,...,k$) are dimensionless numbers. Here $k = n - m$ where m is the number of fundamental units used.

In equation (3), the unit of τ_d and T is time and the unit of K is dependent on the plant input and output. Therefore, the FOPTD model has three variables with only two different units. Hence, there is only one dimensionless number in the model. All dimensionless numbers for the model and the controller are:

$$\frac{\tau_d}{T}, \ \frac{T_i}{\tau_d} \text{ or } \frac{T_i}{T}, \ \frac{T_d}{\tau_d} \text{ or } \frac{T_d}{T} \text{ and } KK_c$$

Based on Buckingham pi-theorem, the PID parameters are obtained from the parameters of the model through determining the second, third and forth dimensionless numbers from the first one, as shown below:

$$KK_c = g_1(\frac{\tau_d}{T})$$
$$\frac{T_i}{\tau_d} = g_2(\frac{\tau_d}{T}) \qquad (10)$$
$$\frac{T_d}{\tau_d} = g_3(\frac{\tau_d}{T})$$

These functions can be driven using numerical optimisation methods such as genetic algorithms. First, for $\frac{\tau_d}{T} = 0.1$, genetic algorithms are used to determine those values of K_c, T_i and T_d which minimise a specific performance index. This step is repeated for $\frac{\tau_d}{T} = 0.2, 0.3, ..., 2$. Therefore, the optimal values of KK_c, $\frac{T_i}{\tau_d}$ and $\frac{T_d}{\tau_d}$ corresponding to the values of $\frac{\tau_d}{T}$ ranging from 0.1 to 2 are determined. Finally, g_1, g_2 and g_3 are driven using curve fitting techniques. The results show that, as Cohen and Coon have suggested, KK_c, $\frac{T_i}{\tau_d}$ and $\frac{T_d}{\tau_d}$ are homographic functions of $\frac{\tau_d}{T}$. Table 1 shows the proposed formulas for different performance indexes.

Table 1. Proposed formulas for different performance indexes

Dimension-less numbers	ISE criterion	IAE criterion	ITAE criterion
KK_c	$\dfrac{0.3\frac{\tau_d}{T}+0.75}{\frac{\tau_d}{T}+0.05}$	$\dfrac{1}{\frac{\tau_d}{T}+0.2}$	$\dfrac{0.8}{\frac{\tau_d}{T}+0.1}$
$\frac{T_i}{\tau_d}$	$\dfrac{2.4}{\frac{\tau_d}{T}+0.4}$	$\dfrac{0.3\frac{\tau_d}{T}+1.2}{\frac{\tau_d}{T}+0.08}$	$0.3+\dfrac{1}{\frac{\tau_d}{T}}$
$\frac{T_d}{\tau_d}$	$\dfrac{1}{90\frac{\tau_d}{T}}$	$\dfrac{1}{90\frac{\tau_d}{T}}$	$\dfrac{0.06}{\frac{\tau_d}{T}+0.04}$

5. SIMULATION RESULTS

In order to compare the performance of the proposed method with the Ziegler-Nichols and Cohen-Coon techniques, three FOPTD models are considered:

$$G_1(s)=\frac{2e^{-0.3s}}{s+1}$$

$$G_2(s)=\frac{5e^{-s}}{1.5s+1}$$

$$G_3(s)=\frac{0.4e^{-1.8s}}{0.9s+1}$$

In the first system the ratio of the time delay to the time constant is relatively small, while the last model involves a system with a relatively long time delay. The PID parameters for these models using the proposed, Ziegler-Nichols and Cohen-Coon formulas are summarized in Table 2. Figures 1-3 show the closed loop step responses resulted from applying these methods to each FOPTD model.

A comparison among the values of performance indexes for the proposed, the Ziegler-Nichols and the Cohen-Coon formulas is presented in Table 3. The table clearly shows that the proposed parameters provide a much better performance for the closed loop system. Moreover, it can be seen from this table that neither Cohen-Coon nor Ziegler-Nichols methods are optimum in terms of any performance index.

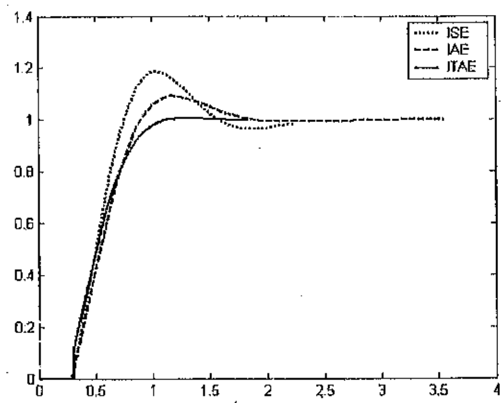

Fig. 1. Closed loop step response resulted from applying proposed PID parameters to $G_1(s)$

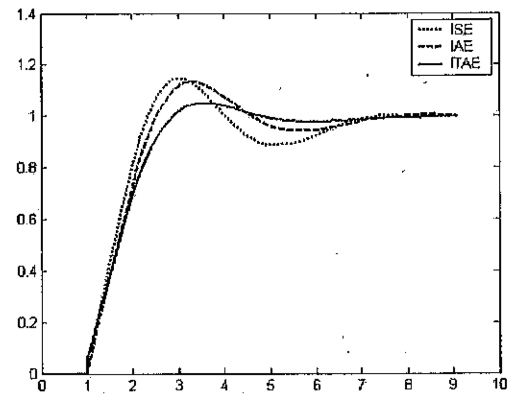

Fig. 2. Closed loop step response resulted from applying proposed PID parameters to $G_2(s)$

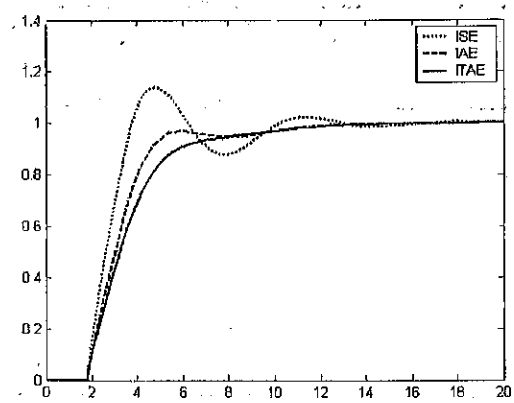

Fig. 3. Closed loop step response resulted from applying proposed PID parameters to $G_3(s)$

6. ROBUSTNESS STUDIES

In order to investigate the robustness of the proposed method in the face of model uncertainties, the model parameters were randomly altered. The nominal parameters of the second FOPTD model are $K=5$, $\tau_d=1$, $T=1.5$. Suppose these parameters are deviated as much as 20% of their nominal values due to uncertainty in the model. The performance indexes for the new model parameters are shown in Table 4. In the second row of this table, for example, τ_d and T have no changes, while K has a reduction of 20%. It can be seen that the worst case is related to an increase of 20% in K and τ_d and a decrease of 20% in T. In this case, the last row of the Table shows that the closed loop step response for the first method of Ziegler-Nichols has an overshoot of more than 100%, while both Cohen-Coon and the second method of Ziegler-Nichols result in unstable closed loop systems. Nevertheless, Figure 4 shows that the closed loop step response for the proposed method is quite satisfactory.

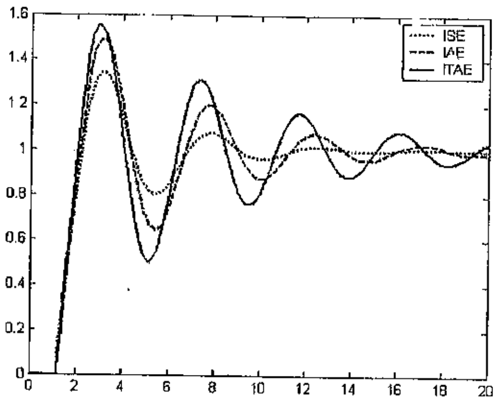

Fig. 4. Closed loop step response resulted from applying proposed PID parameters to $G_2(s)$ with an uncertainty of 20%

7. CONCLUSIONS

PID controllers have been broadly used in process control since the 1940s. Despite a simple structure, they can effectively control a very large group of industrial processes. Furthermore, this controller is often categorised as an almost robust controller; as a result, they may also control uncertain processes. Due to their popularity, many research works have been carried out during the past sixty years to obtain the best formulas for tuning PID parameters, but every method has had a disadvantage or limitation.

In this paper an optimal technique for tuning PID parameters for FOPTD systems was proposed. Dimensional analysis and numerical optimisation methods were used to simplify the procedure of obtaining optimal relations. It was shown that the proposed formulas have a clear advantage to Ziegler-Nichols and Cohen-Coon methods - the most popular techniques in tuning PID controllers. In addition, robustness studies proved the robustness of our method in comparison with two other methods. Our future research is targeted at obtaining optimal formulas for tuning PID controllers for a second order plus time delay model.

8. REFERENCES

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