00016

The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Mechanical Engineering

Study Programme	Master of Energy Management (MEM)
Name of the Examination	Final Examination
Course Code and Title	**DMX5533 Dynamics of Mechanical Systems**
Academic Year	2021/22
Date	02nd February 2023 (Thursday)
Time	09.30 - 12.30 hrs
Duration	**03 hours**

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of in **Six (6)** Pages.
3. Answer **All Questions** in **SECTION A** and any **Four (4)** questions from **SECTION B**
4. This is a Closed Book Test (CBT).
5. **Laplace Transformations** table is given in page **Seven (7).**
5. Answers should be in clear handwriting.
6. Do not use Red color pen.

SECTION A

QUESTION A1 (20 marks)

Figure QA1 (a) shows a simple idealization of a force sensor. Which is used to measure the force **F**, by providing an electrical signal. The electrical signal is proportional to the length *'s'* of the spring.

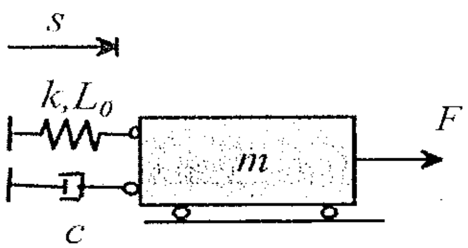

Figure QA1 (a)

At time **t = 0** the system is at rest, and the applied force **F = 0**. When time **t = 1** s an impulsive force of **F = 100N** is applied to the mass "**m**" and the motion of the system is observed. The Figure QA1 (b) shows the variation of spring length "**s**" with time for **0 < t < 5** s.

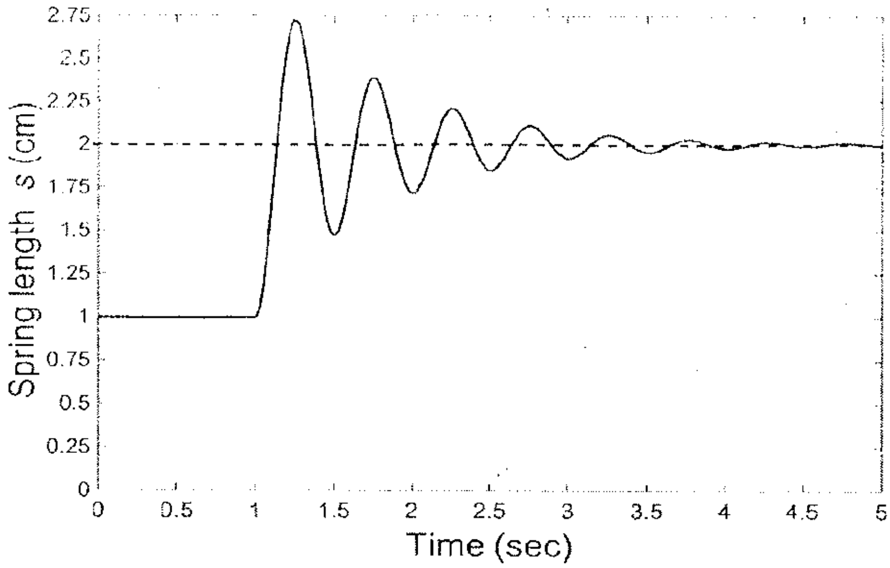

Figure QA1 (b)

(a) Using the graph provided (Figure QA1 (b)), answers the following questions.

i. What is the period (**T**) of vibration?
ii. What is the damped natural frequency (ω_d) of the systems?
iii. Find the log decrement of the vibration (be careful to use the correct origin). - δ
iv. Calculate the damping factor of the system. - ζ
v. Find the undamped natural frequency of the system. - ω_n
vi. Find the un-stretched length of the spring. - L_0
vii. Calculate the spring stiffness. - **k**
viii. What is the value of the mass? - **m**
ix. Calculate the dashpot coefficient. – **c**

(b) The sensor is now used to measure a force that vibrates harmonically $F(t) = F_0 \sin(\omega t)$. The Figure QA1 (c) shows the steady-state variation of the spring length "**s**" with time. Calculate the amplitude of the force.

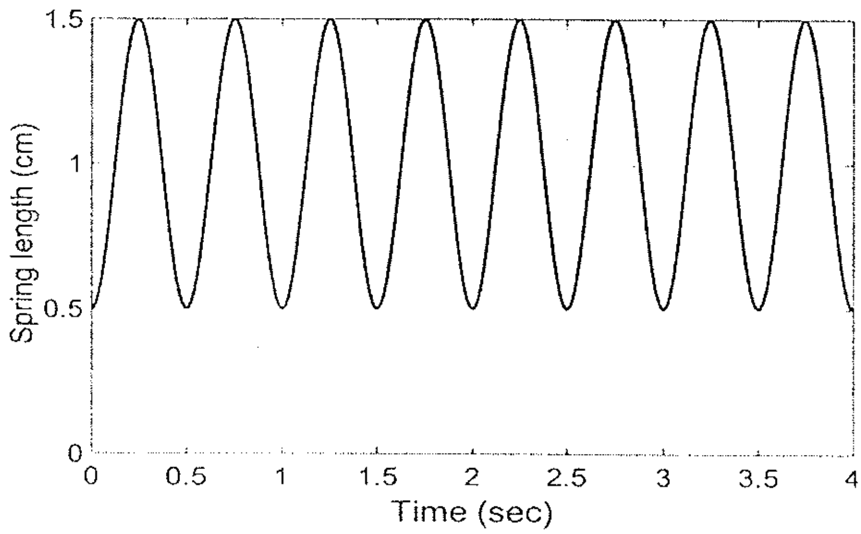

Figure QA1 (c)

Note: The frequency of the force is equal to the natural frequency (the period of vibration is equal to the period in the Figure QA1(b)).

SECTION B

QUESTION 01 (20 marks)

The pistons of the hydraulic system shown in Figure Q B2, are massless and frictionless. The mass **M** is connected to piston **Q** of cross-sectional area **A_2** by a rigid massless rod which slides in the frictionless bearings (Figure Q B2). Piston **P** has a cross-sectional area **A_1**. The region between the pistons **P** and **Q** is filled with an incompressible fluid. If the constriction in the line connecting the two cylinders has a flow resistance **R**, show that the transfer function relating **F** to **Y** is given by,

$$\frac{Y(s)}{F(s)} = \frac{K}{s(s+\tau)} \quad \text{where} \quad K = \frac{A_2}{MA_1} \quad \text{and} \quad \tau = \frac{RA_2^2}{M}$$

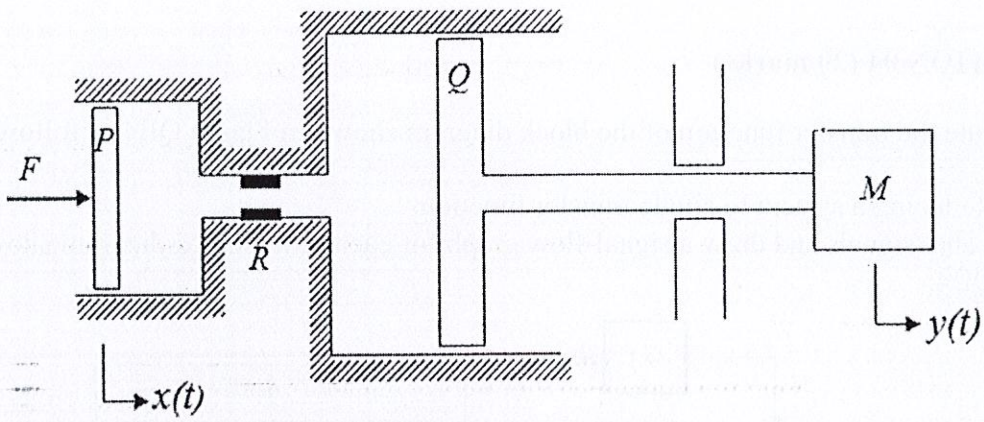

Figure Q B2

QUESTION 02 (20 marks)

(a) A compressor having a rotor of mass **55 kg** with a shaft stiffness of **1.4×10^7 N/m**, with an operation speed of **6000 rpm**, and a measured internal damping providing a damping ratio of **$\beta = 0.05$**. The rotor is assumed to have a worst-case eccentricity of **1000 μm** (e = 0.001 m). Calculate;

i. The critical speed of the rotor.
ii. The radial amplitude at given operating speed.
iii. The whirl amplitude at the system's critical speed.

(b) The clearance specification for the rotor inside the compressor housing limits the whirl amplitude at resonance to be **2 mm**. If the whirl amplitude at operating speed is greater than the allowable clearance,

i. what percent of change in mass is required to redesign this system?
ii. what percent of change in stiffness would result in the same design?
iii. discuss the feasibility of such changes.

QUESTION 03 (20 marks)

A cantilever of length L and mass **m** per unit length has a vertical prop at the free end so that the fixed end and the free end of the cantilever are at the same level. The static deflection y of the propped cantilever at a distance x from the fixed end is given by,

$$y = \frac{mg}{48EI}(2x^4 + 3L^2x^2 - 5Lx^3)$$

where EI is the flexural rigidity of the beam. If the cantilever takes the same form as the static deflection curve when it vibrates transversely, determine the natural frequency of transverse vibration of the propped cantilever in terms of **EI**, **L** and **m**.

QUESTION 04 (20 marks)

Compute the transfer function of the block diagram shown in Figure QB4 in following two cases.

(a) Reducing a system to single transfer function
(b) Label signals and draw a signal-flow graph for each of the block diagram shown in the Figure QB4.

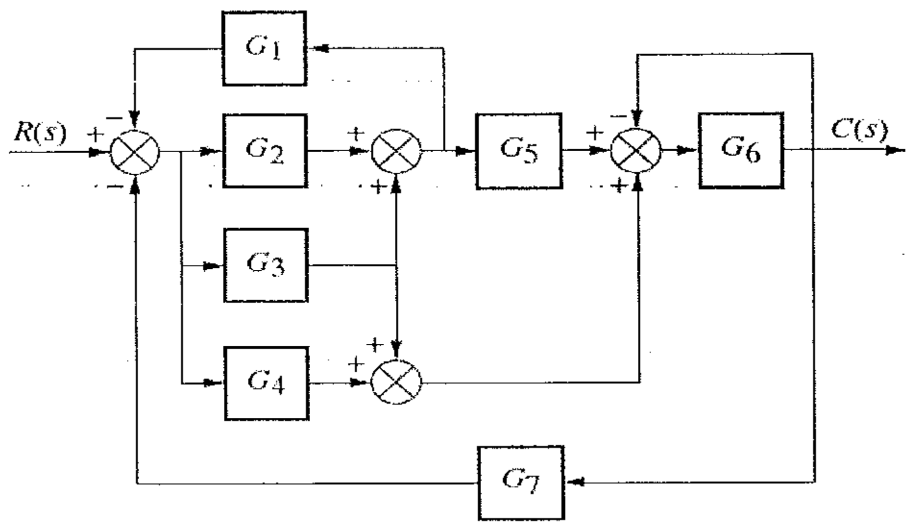

Figure. QB4

QUESTION 05 (20 marks)

The open loop transfer function of a unity feedback system is given by,

$$G(s) = \frac{K}{(s+1)(s^2+8s+20)}$$

Where K is the gain of the controller
(a) Draw the root locus diagram as a function of K and determine the range of K for which the system is stable.
(b) Verify your result using Routh's analysis.

(c) If s = -3 is a closed loop pole of the system when K = 10, derive an expression for the response of the system for a unit impulse input when K = 10. Assume all initial conditions to be zero.

QUESTION 06 (20 marks)

Figure QB6 shows a control system. Draw a Bode diagram of the open loop transfer function and determine the value of the gain **K** such that the phase margin is **50°**. What is the gain margin of this system with this gain **K**?

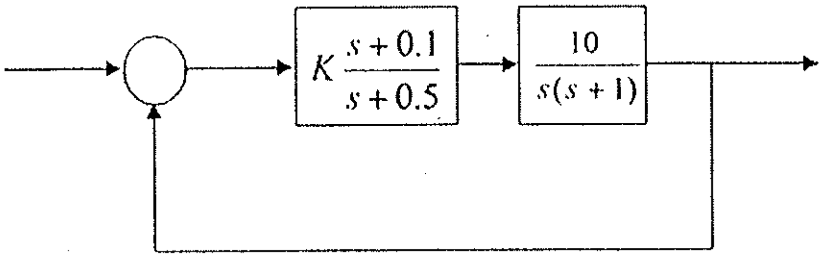

Figure. QB6

LAPLACE TRANSFORMS

TIME FUNCTION $f(t)$	LAPLACE TRANSFORM $F(s)$
Unit Impulse $\delta(t)$	1
Unit step	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
$\frac{df(t)}{dt}$	$sF(s)-f(0)$
$\frac{d^n f(t)}{dt^n}$	$s^n F(s) - s^{n-1} f(0) - s^{n-2} \frac{df(0)}{dt} \ldots - \frac{d^{n-1} f(0)}{dt^{n-1}}$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$