The Open University of Sri Lanka
Department of Electrical and Computer Engineering
ECX6241 – Field Theory
Final Examination – 2016/2017
Bachelor of Technology Honours in Engineering

Date: 2017-11-22 Time: 1330-1630

Answer **Only five questions** by selecting **two from Section A, two from Section B** and **one from Section C**. This paper has three pages. All the notations have their usual meaning. Assume any missing parameter with suitable values.

Section A

Select **only two** questions from this section. (15 Marks for each)

Q1.

(a) Define scalar and vector fields by giving suitable example for each. [3]

(b) For a vector field $\boldsymbol{A}$, show explicitly that $\nabla \cdot \nabla \times \boldsymbol{A} = 0$ [4]

(c) The electric field at point P expressed in cylindrical coordinates is given by $\boldsymbol{E} = 16r^3 \sin\phi\, \boldsymbol{a}_r + 3r^2 \cos\phi\, \boldsymbol{a}_\phi$. Determine the divergence of $\boldsymbol{E}$, if the location of the point P is given by (1,2,3) in Cartesian coordinates. [8]

Q2.

(a) Classify vector fields by using its divergence and curl. [3]

(b) State the divergence theorem. [3]

(c) If vector $\boldsymbol{D} = \frac{5\rho^2}{4}\boldsymbol{a}_\rho$ is given in spherical coordinates, verify both sides of the divergence theorem for volume enclosed by $\rho = 1$ and $\rho = 2$ [9]

Q3.

(a) Determine the Laplacian of the scalar field $F = 10\rho \sin^2\theta \cos\phi$ [4]

(b) State the Stoke's theorem. [3]

(b) If $\boldsymbol{A} = r\sin\phi\, \boldsymbol{a}_r + r^2\, \boldsymbol{a}_\phi$ is given in cylindrical coordinates, verify the Stoke's theorem for contour shown in the figure Q3. [8]

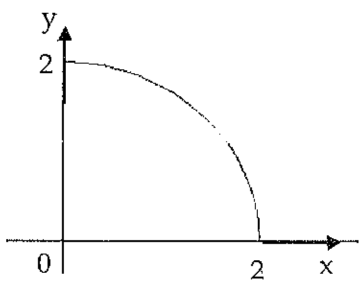

Figure Q3

Section B

Select **only two** questions from this section. (20 Marks for each)

Q4.

(a) State the Coulomb's law. [3]

(b) Obtain Coulomb's law from Gauss's Theorem. [5]

(b) Three identical small spheres of mass m are suspended from a common point by threads of negligible mass and equal length l. A charge Q is divided equally among the spheres and they come to equilibrium at the corners of a horizontal equilateral triangle whose sides are d. Show that [12]

$$Q^2 = {12\pi\varepsilon_0 mgd^3}\Big/{\left(l^2 - \frac{d^2}{3}\right)}$$

Q5.

(a) State the uniqueness theorem. [3]

(b) A spherical volume charge density distribution is given by

$$\rho = \begin{cases} \rho_0\left[1 - {r^2}/{a^2}\right] & ; r \leq a \\ 0 & ; r > a \end{cases}$$

i. Calculate the total charge Q [6]

ii. Determine $\boldsymbol{E}$ for $0 < r \leq a$ and $r > a$ [6]

iii. Show that the maximum of $\boldsymbol{E}$ is at $r = 0.745\ a$ [5]

Q6.

(a) State and explain the Ampere's law. [4]

(b) Distinguish between magnetic vector potential and magnetic scalar potential. [4]

(c) Prove that the magnetic scalar potential at point $(0,0,z)$ due to a circular loop of current I of radius a on $z = 0$ plane is [12]

$$V_m = \frac{I}{2}\left[1 - {z}\Big/{\sqrt{z^2 + a^2}}\right]$$

Section C

Select **only one** question from this section. (30 Marks)

Q7.

(a) Explain the "Electromagnetic induction" in electromagnetism theory. [5]

(b) Show that the continuity equation $\nabla \cdot J + \frac{\partial \rho}{\partial t} = 0$ is continued in Maxwell's equation. [10]

(c) An electric field vector $\boldsymbol{E}$ of an electromagnetic wave in free space is given by $E_x = E_y = 0$ and $E_z = Ae^{j\omega(t - z/v)}$. Using Maxwell's equation for free space condition, determine an expression for components of the magnetic field vector $\boldsymbol{H}$. [15]

Q8.

(a) Discuss the wave propagation in [6]

i. lossy dielectric

ii. conductor

(b) For a lossless dielectric medium, $\eta = 60\pi, \mu_r = 1$ and

$\boldsymbol{H} = -0.1\cos(\omega t - z)\,\boldsymbol{a}_x + +0.5\sin(\omega t - z)\,\boldsymbol{a}_y\ {}^{A}/_{m}$. Determine ε_r, ω and $\boldsymbol{E}$. [18]

(c) Determine the Poynting vector. [6]

-end-

Note:

Cylindrical Coordinates

$$\nabla f = \frac{\partial f}{\partial \rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial f}{\partial \varphi}\hat{\varphi} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$$

$$\nabla \cdot \boldsymbol{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \boldsymbol{A} = \left(\frac{1}{\rho}\frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z}\right)\hat{\rho} + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right)\hat{\varphi} + \frac{1}{\rho}\left(\frac{\partial}{\partial \rho}(\rho A_\varphi) - \frac{\partial A_\rho}{\partial \varphi}\right)\hat{\mathbf{z}}$$

$$\nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \varphi^2} + \frac{\partial^2 f}{\partial z^2}$$

Spherical Coordinates

$$\nabla f = \frac{\partial f}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}\hat{\boldsymbol{\varphi}},$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 A_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial \varphi},$$

$$\nabla \times \mathbf{A} = \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial \theta}(A_\varphi \sin\theta) - \frac{\partial A_\theta}{\partial \varphi}\right)\hat{\mathbf{r}} + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \varphi} - \frac{\partial}{\partial r}(rA_\varphi)\right)\hat{\boldsymbol{\theta}} + \frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial \theta}\right)\hat{\boldsymbol{\varphi}},$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \varphi^2}$$

$$= \left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}\right)f + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial}{\partial \theta}\right)f + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial \varphi^2}f.$$