THE OPEN UNIVERSITY OF SRI LANKA
Faculty of Engineering Technology
Department of Mathematics and Philosophy of Engineering

Bachelor of Industrial Studies Honors

0725

Final Examination (2021/2022)
MHZ5570: Quantitative Techniques

Date: 17/02/2023 **Time: 14:00-17:00 hours**

Instructions:

- This paper consists of Seven (7) questions in Five (5) pages.
- The first question is compulsory and answer any Four (4) other questions.
- Relevant equations are provided.
- State any assumptions you required and show all your workings.
- This is a closed book test and do not use red color pen.

1. (a) Solve the following equations.
   i. $\frac{2^{3x+1}}{4^x} = 64$ (*Marks 10*)
   ii. $2\log_2 x - \log_2(x-2) = 3$ (*Marks 10*)

   (b) Find the point that the graph of function $y = x^2 + 4x - 5$ cross the $y-$axis. (*Marks 5*)

   (c) Let $A = \begin{pmatrix} 0 & 2 & 1 \\ -2 & 0 & 3 \\ -1 & -3 & 0 \end{pmatrix}$. Then find the following matrices.
   i. $A + A^T$ (*Marks 10*)
   ii. $A^3 + 14A$ (*Marks 15*)

   (d) Determine the rate of change of function $f(x) = \log_e(3x) + 5e^{-3x} - \frac{1}{2x^2}$ at $x = 1$. (*Marks 10*)

   (e) An industrialist has three machines that are used in the manufacture of three different products. To fully unitize the machines, they are to be kept operating for 8 hours per a day. The number of hours each machine is used in the production of one unit of each of the three products is given by

	$product-1$	$product-2$	$product-3$
$machine-1$	1	2	1
$machine-2$	2	0	1
$machine-3$	1	2	3

Write the required equations to find the number of units of each of the three products that would be produced in one day under the assumption that each machine is used for the full 8 hours per day. Clearly state your variables. Do not solve the equations. (*Marks 15*)

(f) Find the minimum value of $Z = x + y$ subjected to constraints $x + 3y \geq 3$, $x, y \geq 0$. (*Marks 15*)

(g) A company uses 7300 units for an item annually, delivery lead time is 8 days. Find the reorder point, in number of units, to order optimum quantity. Assume that a year consists with 365 days. (*Marks 10*)

2. (a) Determine the inverse of matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -2 & -3 \\ 1 & -1 & -1 \end{pmatrix}$. (*Marks 55*)

(b) Hence solve the following system of equations. (*Marks 45*)

$$x + y + z = -2$$
$$x - 2y - 3z = 4$$
$$x - y - z = 12.$$

3. Let $y = 2t^4 - 8t^3$ for $t \in \mathbb{R}$.

(a) Find $\frac{d^2y}{dt^2}$. (*Marks 25*)

(b) Find the maximum/minimum/inflection points of y. Justify your answer. (*Marks 35*)

(c) Sketch the curve y. (*Marks 40*)

4. (a) A company manufactures shirts and pairs of pants and the profit per unit sold is Rs.300 and Rs.500 respectively. Each product has to be assembled on a particular machine, each unit of shirts takes 12 minutes of assembly time and each unit of pair of pants takes 25 minutes of assembly time. The company estimates that the machine used for assembly has an effective working week of only 1800 minutes (due

to maintenance/breakdown). Technological constraints mean that it should produce at least 50 units of shits and at least 20 units of pair of pants per week. The company wants to maximize the profit. Formulate a linear programming problem according to the given details. *(Marks 20)*

(b) A company manufactures two products A and B. Each unit of product A requires 1 unit of raw materials and 4 units of machine hours, and each unit of product B requires 1 units of raw materials and 2 units of machine hours. The company has a weekly supply of 12 units of raw materials and 32 units of machine hours.
The company sells a unit of product A and product B for Rs.400 and Rs.300 respectively and it wishes to maximize its revenue. Consider the following formulation of the problem.

$$\begin{aligned} \text{maximize: } & z = 400x + 300y \\ \text{subject to } & x + y \leq 12 \\ & 4x + 2y \leq 32 \\ & x \geq 0, y \geq 0. \end{aligned}$$

Where, x- Number of items produced in product A
y- Number of items produced in product B
z- Revenue get from selling product A and Product B.

i. Explain why is this a linear programming problem? *(Marks 05)*

ii. If the simplex method is supposed to be used to solve the problem, write down the problem in canonical form. *(Marks 15)*

iii. Write down the initial tableau of the problem in order to follow the simplex method. *(Marks 20)*

iv. If the following tableau gives an intermediate step of the simplex method while solving the problem, continue the procedure till you get the final tableau. *(Marks 30)*

	x	y	s_1	s_2	z	RHS
s_1	0	$\frac{1}{2}$	1	$-\frac{1}{4}$	0	4
x	1	$\frac{1}{2}$	0	$\frac{1}{4}$	0	8
z	0	-100	0	100	1	3200

v. Write down the optimal production plan which will maximize the revenue. *(Marks 10)*

5. A firm produces three products. These products are proceeded on three different machines. The time required to manufacture one units of each of three product and the manufacturing capacity of three machines are given in following table.

Machine	**Time per unit(Hours)**			**Machine Capacity (Hours per week)**
	Product 1	Product 2	Product 3	
M1	4	1	1	30
M2	2	3	1	60
M3	1	2	3	40

The profit per unit for product 1, product 2 and product 3 are $Rs.30, Rs.20$ and $Rs.10$ respectively. It is assumed that all the products produced are consumed in the market.

(a) Identify the decision variables. *(Marks 15)*

(b) Formulate a mathematical model(Linear Programming Model) to estimate the production structure which will maximize weekly profit. *(Marks 25)*

(c) Convert the linear programming problem in part (b) to standard form of the linear programming model. *(Marks15)*

(d) Write down the basic solution of the initial tabular of the simplex method which use to solve the linear programming problem in part (c). *(Marks 10)*

(e) Solve the linear programming problem in part (c) using simplex method. *(Marks 35)*

6. (a) Discuss the importance of inventory maintenance in the apparel industry. *(Marks 20)*

(b) A company requires 2500 units of a certain item per year. The ordering cost per order and the inventory holding charges per unit per year are Rs. 1250 and Rs. 400 respectively.

i. Find the optimal number of units which should order per order by the company to minimize the total inventory cost? *(Marks 20)*

ii. State all the assumptions that you made in part (a) if there are any. *(Marks 10)*

iii. Find the maximum level of inventory, that company will maintain by minimizing inventory cost. *(Marks 10)*

iv. Assuming that there are 360 days in a year, find the optimal cycle time in days. *(Marks 10)*

v. Find the optimal number of of orders per year that should minimize the total cost of inventory. *(Marks 10)*

vi. What will be the total cost of carrying and ordering inventories per cycle when order the optimal order quantity? *(Marks 20)*

7. (a) Specify the advantages and disadvantages of holding inventory in manufacturing business. *(Marks 20)*

(b) A manufacturer can produce 120 T-shirts per day and he receives an order of 30 T-shirts per day for a customer . The cost of holding a T-shirt in inventory is Rs. 40 per month and the production(set up) cost per production(set up) is Rs. 2000. Answer to following questions by assuming that there are 30 days per month.

i. Find the optimal production quantity of a single production which will minimize the total inventory cost. *(Marks 25)*

ii. Find the period in days where the both production and selling (purchase by school) will be. *(Marks 15)*

iii. Find the period in days where only the selling will be. *(Marks 10)*

iv. How frequently the production run have to be made per month to satisfy the demand? *(Marks 10)*

v. Due to the limited availability of machines, if the amount of production has to be limited to 200 T shirts per run, find the minimum total cost of an inventory. *(Marks 20)*

End.

Equations

$$Q^* = \sqrt{\frac{2C_0D}{C_c}}$$

$$T = C_o\frac{D}{Q} + C_c\frac{Q}{2} + C_pQ$$

$$T = C_o\frac{D}{Q} + C_c\frac{Q}{2}$$

$$Q^* = \sqrt{\frac{2C_0D}{C_c}}\sqrt{\frac{C_s + C_c}{C_s}}$$

$$S^* = \sqrt{\frac{2C_0D}{C_s}}\sqrt{\frac{C_c}{C_s + C_c}}$$

$$Q^* - S^* = \sqrt{\frac{2C_0D}{C_s}}\sqrt{\frac{C_c}{C_s + C_c}}$$

$$Q^* = \sqrt{\frac{2C_0r}{C_c\left(1 - \frac{r}{p}\right)}}$$

$$t_1^* = \frac{Q^*}{p}$$

$$t_2^* = \frac{Q^*}{r}\left(1 - \frac{r}{p}\right)$$