THE OPEN UNIVERSITY OF SRI LANKA
Faculty of Engineering Technology
Department of Mathematics and Philosophy of Engineering
Bachelor of Industrial Studies Honors

Final Examination (2021/2022)
TAZ5550: Quantitative Techniques 045

Date: 17/02/2023 **Time: 14:00-17:00 hours**

Instructions:

- This paper consists of Seven (7) questions in Four (4) pages.
- The first question is compulsory and answer any Four (4) other questions.
- State any assumptions you required and show all your workings.
- This is a closed book test and do not use red color pen.

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1. (a) Solve the following equations.

   i. $\frac{2^{3x+1}}{4^x} = 64$ *(Marks 10)*

   ii. $2\log_2 x - \log_2(x-2) = 3$. *(Marks 10)*

   (b) Find the point that the graph of function $y = x^2 + 4x - 5$ cross the y-axis. *(Marks 5)*

   (c) Let $A = \begin{pmatrix} 0 & 2 & 1 \\ -2 & 0 & 3 \\ -1 & -3 & 0 \end{pmatrix}$. Then find the following matrices.

   i. $A + A^T$ *(Marks 10)*

   ii. $A^3 + 14A$ *(Marks 15)*

   (d) Determine the rate of change of function $f(x) = \log_e(3x) + 5e^{-3x} - \frac{1}{2x^2}$ at $x = 1$. *(Marks 10)*

   (e) An industrialist has three machines that are used in the manufacture of three different products. To fully unitize the machines, they are to be kept operating for 8 hours per a day. The number of hours each machine is used in the production of one unit of each of the three products is given by

	product − 1	*product* − 2	*product* − 3
machine − 1	1	2	1
machine − 2	2	0	1
machine − 3	1	2	3

Write the required equations to find the number of units of each of the three products that would be produced in one day under the assumption that each machine is used for the full 8 hours per day. Clearly state your variables. Do not solve the equations. *(Marks 15)*

(f) Find the minimum value of $Z = x + y$ subjected to constraints $x + 3y \geq 3$, $x, y \geq 0$. *(Marks 15)*

(g) If the objective function $Z = ax + by$ of a linear programming problem, has the same maximum value on two corner points of the feasible region, then what you can say about the optimal solution of this linear programming problem? *(Marks 10)*

2. (a) Determine the inverse of matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -2 & -3 \\ 1 & -1 & -1 \end{pmatrix}$. *(Marks 55)*

(b) Hence solve the following system of equations. *(Marks 45)*

$$x + y + z = -2$$
$$x - 2y - 3z = 4$$
$$x - y - z = 12.$$

3. Let $y = 2t^4 - 8t^3$ for $t \in \mathbb{R}$.

(a) Find $\dfrac{d^2y}{dt^2}$. *(Marks 25)*

(b) Find the maximum/minimum/inflection points of y. Justify your answer. *(Marks 35)*

(c) Sketch the curve y. *(Marks 40)*

4. The manufacturing cost of an item consists of three components: fixed cost, material cost and labour cost, with the fixed cost and the material cost set Rs.1,200 and Rs.30 per item respectively and the labour cost is given as Rs. $\dfrac{x^2}{100}$ for x items produced. The profit from producing x number of items is Rs.$\dfrac{x^3}{10000} + 2x$.

(a) Write the total cost for producing x items. *(Marks 20)*

(b) Find the marginal cost of the product when producing 1000 items. Interpret your answer. *(Marks 30)*

(c) Find the revenue of the company assuming all the produced items will be sold. *(Marks 20)*

(d) Find the marginal revenue of the product with 1000 products. Interpret your answer. *(Marks 30)*

5. (a) A company manufactures shirts and pairs of pants and the profit per unit sold is Rs.300 and Rs.500 respectively. Each product has to be assembled on a particular machine, each unit of shirts takes 12 minutes of assembly time and each unit of pair of pants takes 25 minutes of assembly time. The company estimates that the machine used for assembly has an effective working week of only 1800 minutes (due to maintenance/breakdown). Technological constraints mean that it should produce at least 50 units of shits and at least 20 units of pair of pants per week. The company wants to maximize the profit. Formulate a linear programming problem according to the given details. *(Marks 20)*

(b) A company manufactures two products A and B. Each unit of product A requires 1 unit of raw materials and 4 units of machine hours, and each unit of product B requires 1 units of raw materials and 2 units of machine hours. The company has a weekly supply of 12 units of raw materials and 32 units of machine hours.
The company sells a unit of product A and product B for Rs.400 and Rs.300 respectively and it wishes to maximize its revenue. Consider the following formulation of the problem.

$$\text{maximize: } z = 400x + 300y$$
$$\text{subject to } x + y \leq 12$$
$$4x + 2y \leq 32$$
$$x \geq 0, y \geq 0.$$

Where, x- Number of items produced in product A

y- Number of items produced in product B

z- Revenue get from selling product A and Product B.

i. Explain why is this a linear programming problem? *(Marks 05)*

ii. If the simplex method is supposed to be used to solve the problem, write down the problem in canonical form. *(Marks 15)*

iii. Write down the initial tableau of the problem in order to follow the simplex method. *(Marks 20)*

iv. If the following tableau gives an intermediate step of the simplex method while solving the problem, continue the procedure till you get the final tableau. *(Marks 30)*

	x	y	s_1	s_2	z	RHS
s_1	0	$\frac{1}{2}$	1	$-\frac{1}{4}$	0	4
x	1	$\frac{1}{2}$	0	$\frac{1}{4}$	0	8
z	0	-100	0	100	1	3200

v. Write down the optimal production plan which will maximize the revenue. *(Marks 10)*

6. A firm produces three products. These products are proceeded on three different machines. The time required to manufacture one units of each of three product and the manufacturing capacity of three machines are given in following table.

Machine	**Time per unit(Hours)**			**Machine Capacity (Hours per week)**
	Product 1	Product 2	Product 3	
M1	4	1	1	30
M2	2	3	1	60
M3	1	2	3	40

The profit per unit for product 1, product 2 and product 3 are $Rs.30, Rs.20$ and $Rs.10$ respectively. It is assumed that all the products produced are consumed in the market.

(a) State the decision variables. *(Marks 15)*

(b) Formulate a mathematical model(Linear Programming Model) to estimate the production structure which will maximize weekly profit. *(Marks 25)*

(c) Convert the linear programming problem in part (b) to standard form of the linear programming model. *(Marks15)*

(d) Write down the basic solution of the initial tabular of the simplex method which use to solve the linear programming problem in part (c). *(Marks 10)*

(e) Solve the linear programming problem in part (c) using simplex method. *(Marks 35)*

7. A manufacturing firm has to manufacture 15000 packages of certain product per week. This firm has 20 small machines where each machine can produced 1000 packages per week and 10 large machines where each of them will produced 2000 packages per week. There is a policy used in this firm that they will use at least as many possible small machines as large machines. Also a large machine costs Rs. 8000 per week and a small machines costs Rs. 6000 per week averagely.

(a) Formulate the linear programming problem according to the given details. Clearly defined decision variables, labeled objective function and all necessary constraints needed to be included to the answer. *(Marks 40)*

(b) Use the graphical method to find the optimal solution for the linear programming problem formulated in part-(a). *(Marks 60)*

End.