The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Mechanical Engineering

Study Programme	: Bachelor of Technology Honours in Engineering
Name of the Examination	: Final Examination
Course Code and Title	**: DMX6304**
	Computational Fluid Dynamics
Academic Year	: 2021/22
Date	: 13th Monday , February 2023
Time	: 1330-1630hrs

General Instructions

1. Read all instructions carefully before answering the questions.
2. Answer all **five (05)** Questions. All questions carry equal marks.
3. This is a Closed Book Test (CBT).
4. Answers should be in clear handwriting.
5. Do not use Red colour pen.

Q1 (a) Explain why CFD is both a powerful and a risky tool. (8 marks)

(b) Define the below terms. (12 marks)

1. Convergence.
2. Boundness
3. Conservation
4. Stability

Q2 A partial differential equation is given below.

$$A\frac{\partial^2 U}{\partial x^2} + B\frac{\partial^2 U}{\partial x \partial y} + C\frac{\partial^2 U}{\partial y^2} + D\frac{\partial U}{\partial x} + E\frac{\partial U}{\partial y} + FU + G = 0$$

Derive the conditions that this equation should satisfy if it is,

(a) Elliptic (5 marks)

(b) Parabolic (5 marks)

(c) Hyperbolic (5 marks)

State whether the equation given below is Elliptic, Parabolic or Hyperbolic. (5 marks)

$$\frac{\partial U^2}{\partial x^2} + \frac{\partial U^2}{\partial y^2} = 0$$

Q3 (a) Explain the three Discretization Approaches in Computational Fluid Mechanics. (5 marks)

(b) A Taylor Series of function $\emptyset(x)$ about the point x_i is given by,

$$\emptyset(x) = \emptyset(x_i) + (x - x_i)\left(\frac{\partial \emptyset}{\partial x}\right)_i + \frac{(x - x_i)^2}{2!}\left(\frac{\partial^2 \emptyset}{\partial x^2}\right)_i + \frac{(x - x_i)^3}{3!}\left(\frac{\partial^3 \emptyset}{\partial x^3}\right)_i + \cdots + \frac{(x - x_i)^n}{n!}\left(\frac{\partial^n \emptyset}{\partial x^n}\right)_i + H$$

The term H refers to the higher order terms in the expansion. Obtain an expression for approximation of first order derivative with:

(a) Forward difference scheme (5 marks)
(b) Backward difference scheme (5 marks)
(c) Central difference scheme (5 marks)

Q4 (a) Explain the iterative solution methods that can be used for solving elliptic partial differential equations. (5 marks)

(b) The Laplace equation which is one of the typical elliptic equations with usual notation is given by:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

(a) Discretize the Laplace equation using five-point finite difference scheme. (5 marks)
(b) The Laplace equation is needed to be solved for the 2-dimensional cartesian grid shown in figure Q4. Obtain the system of equations in matrix form. (10 marks)

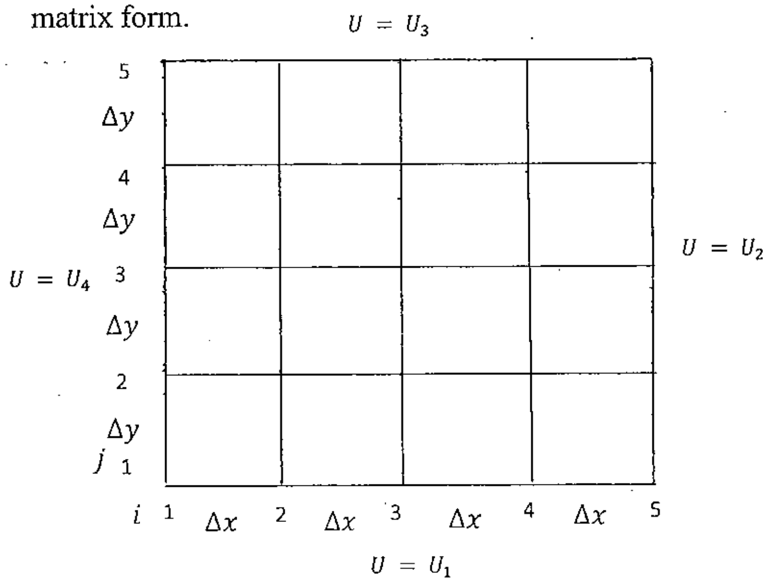
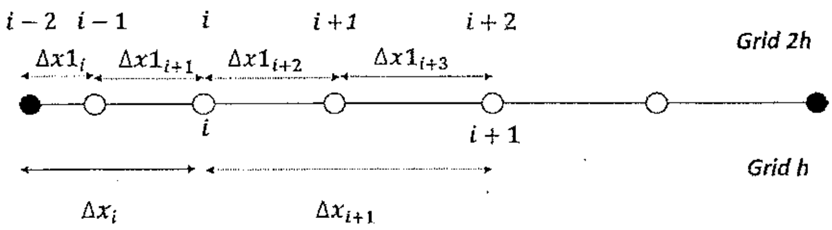


Figure Q4

Q5 (a) Figure Q5 shows a grid system h and 2h formulated to test how the grid refinement affects the truncation error in computational fluid dynamics. The grid expansion ratio for *grid h* is r_e and the same for *grid 2h* is r_a.



(10 marks)

(a) Show that $r_a = \sqrt{r_e}$ (10 marks)

(b) If the leading truncation error term for the central difference scheme for *grid h* is given by:

$$\epsilon_\tau \approx \frac{(1 - r_e)\Delta x_i}{2}\left(\frac{\partial^2 \emptyset}{\partial x^2}\right)_{x_i}$$

Show that the error ratio $r_\tau = \frac{(1-r_a^2)(r_a+1)}{(1-r_a)r_a}$

END