



Study Programme	: Bachelor of Technology Honours in Engineering
Name of the Examination	: Final Examination
Course Code and Title	: <b>DMX6573 Advanced Control Engineering</b>
Academic Year	: 2021/2022
Date	: 28 <sup>th</sup> February 2023
Time	: 13.30-16.30
Duration	: <b>3 hours</b>

### General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **Six (6)** questions in **Six (6)** pages.
3. Answer **Question 1**, which is **compulsory**, and **three** others.
4. Question 01 carries 40 marks and others 20 marks each.
5. Answer for each question should commence from a new page.
6. This is a Closed Book Test (CBT).
7. Answers should be in clear handwriting.
8. Do not use Red colour pen.

### Question 01

#### Active Suspension System

A vehicle's suspension system typically consists of springs and shock absorbers that help to isolate the vehicle chassis and occupants from sudden vertical displacements of the wheel assemblies during driving. A well-tuned suspension system is important for the comfort and safety of the vehicle occupants as well as the long-term durability of the vehicle's electronic and mechanical components.

The suspension systems in most vehicles on the road today are passive. The chassis of the vehicle is attached to the axles or wheel assemblies through coil springs or leaf springs that help to protect the chassis from sudden vertical forces applied to the wheels. The shock

absorbers help to dissipate the energy applied to the springs and damp the oscillations that would normally occur when a brief excitation is applied to a mass-spring system.

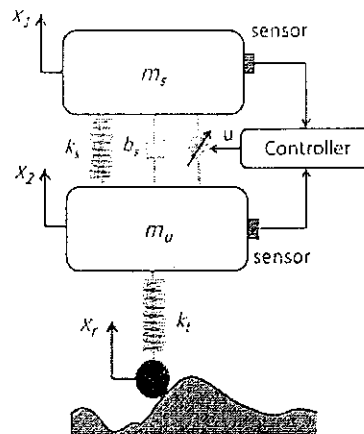


Figure Q1

Active suspension systems sense the forces being applied to the wheels and constantly adjust the mechanical connections between the chassis and wheel assemblies to keep the chassis level and/or optimally absorb the energy associated with the vertical motion of the wheels. Additionally, with the advent of increased computer control, various options of suspension travel and response can be adjusted by the driver while driving.

As shown in Figure Q1,  $m_s$  denotes the mass on the spring and  $m_u$  denotes the mass under the spring;  $x_r$  denotes the road disturbance excitation,  $x_2$  denotes the vertical displacement of the mass under the spring, and  $x_1$  denotes the vertical displacement of the mass on the spring;  $b_s$  denotes the suspension equivalent damping;  $k_t$  is the tire equivalent stiffness and  $k_s$  is the suspension stiffness; and  $u$  is the actuator active control force.

- Select suitable the sensors and actuators considered in the active suspension system shown in figure Q1. Explain the operation of each sensor and actuator. [4]
- Derive the state equation of the active suspension model shown in figure Q1. [10]
- Develop the control system block diagram to control active suspension system. You should clearly indicate inputs, outputs, feedbacks and necessary components. [6]
- State the condition for system observability and check the observability of the system for linearized state equations. [10]

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 \\ 5 & 0 & 2 \\ 0 & 3 & 4 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} U$$

$$Y = [1 \ 0 \ 0]X \quad [10]$$

- Design a full order observer for system equations in above question (d) such that the observer poles are located at  $-1 \pm j6$ ,  $-10$ . Give all the relevant observer equations. [10]

Question 02

a) A function  $V(x): \mathbb{R}^n \rightarrow \mathbb{R}$  is described as positive definite. Explain what this means.

[5]

b) A function  $V(x)$  is said to be radially unbounded. Explain what this means and why it is important in the study of stability using Lyapunov functions.

[5]

c) A mass-spring-damper system with nonlinear damping is described by the following differential equations:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -x_1 - x_2 x_1^2 + x_2$$

1) Linearize this system about the point  $x_1 = x_2 = 0$  and find the Jacobian matrix.

$$J_{(F)}(x_1, x_2)$$

[5]

2) Determine whether a symmetric positive definite solution exists to the following Lyapunov equation, where "A" is Jacobian matrix answer to part 1)

$$A'P + PA = -I$$

If it is solvable, find the matrix P. If not, explain your answer.

[5]

Question 03

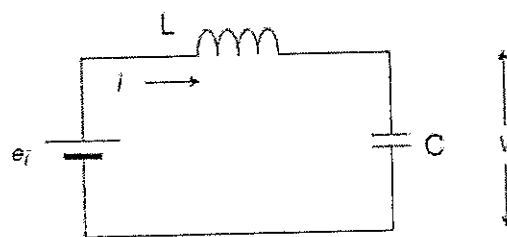


Figure Q3

Consider the electrical circuit diagram shown in Figure Q3;

a) Derive the system equations of above electrical circuit.

[3]

b) Determine the state space model of the system.

[4]

c) Determine the system stability by using Lyapunov second method (Direct Method)

[6]

d) Consider a dynamic system that is represented by the following input-output differential equation:

$$\ddot{y} + 10\dot{y} + 65y + 100y = \ddot{u} + 5\dot{u} + 10u + 7u$$

Draw a simulation block diagram for this system using integration, gain blocks and summing points only. ( $y$  denotes output and  $u$  denotes input) [7]

#### Question 04

a) Describe the key difference between a Mamdani controller and a Takagi-Sugeno Controller. [3]

b) A fuzzy set A and B in the universe R whose membership function is given by

$$\begin{aligned} \mu_A(x) &= 1 - (0.5|x - 3|) \text{ For } 1 \leq x \leq 5 = 0 \text{ otherwise} \\ \mu_B(x) &= 1 - (0.5|x - 4|) \text{ For } 2 \leq x \leq 6 = 0 \text{ otherwise} \end{aligned}$$

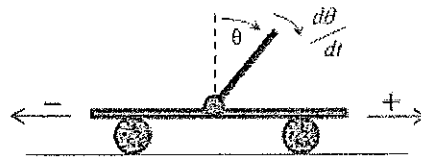
1) Sketch and describe the membership functions  $\mu_A(x)$  and  $\mu_B(x)$ .

2) Determine  $A \cup B$

3) Determine  $A \cap B$

4) What is  $\alpha$ -cut of B for  $\alpha=0.5$  [5]

c) Figure Q4 shows two input, single output Pole-Balancing control system: The input variables,  $\theta$  and  $d\theta$  and  $\eta$  is the output variable for actuator.



The variables belong to the universes of discourse  $X_1$ ,  $X_2$  and  $Y$  respectively, such that  $\theta \in X_1 = [-2, 2]$ ,  $d\theta \in X_2 = [-5, 5]$  and  $\eta \in Y = [-16, 16]$ . The fuzzy partitions of the sets  $X_1$ ,  $X_2$  and  $Y$  are given as

For X1

$$\begin{aligned} \text{NEG}_{\mu(\theta)} &= \begin{cases} 1 & -2 \geq \theta \\ -0.5\theta & -2 < \theta < 0 \end{cases} \\ \text{ZERO}_{\mu(\theta)} &= \begin{cases} 0.5\theta + 1 & -2 < \theta < 0 \\ -0.5\theta + 1 & 0 < \theta < 2 \end{cases} \\ \text{POS}_{\mu(\theta)} &= \begin{cases} 0.5\theta & 0 < \theta < 2 \\ 1 & 2 \leq \theta \end{cases} \end{aligned}$$

For X2

$$\begin{aligned} \text{NEG}_{\mu(d\theta)} &= \begin{cases} 1 & -5 \geq d\theta \\ -0.2d\theta & -5 < d\theta < 0 \end{cases} \\ \text{ZERO}_{\mu(d\theta)} &= \begin{cases} 0.2d\theta + 1 & -5 < d\theta < 0 \\ -0.2d\theta + 1 & 0 < d\theta < 5 \end{cases} \\ \text{POS}_{\mu(d\theta)} &= \begin{cases} 0.2d\theta & 0 < d\theta < 5 \\ 1 & 5 \leq d\theta \end{cases} \end{aligned}$$

For Y

$$\begin{aligned} \text{BN}_{\mu(y)} &= \begin{cases} 1 & -16 \geq y \\ -0.125y - 1 & -16 < y < -8 \end{cases} \\ \text{N}_{\mu(y)} &= \begin{cases} 0.125y + 2 & -16 < y < -8 \\ -0.125y & -8 < y < 0 \end{cases} \\ \text{Z}_{\mu(y)} &= \begin{cases} 0.125y + 1 & -8 < y < 0 \\ -0.125y + 1 & 0 < y < 8 \end{cases} \\ \text{P}_{\mu(y)} &= \begin{cases} 0.125y & 0 < y < 8 \\ -0.125y + 2 & 8 < y < 16 \end{cases} \\ \text{BP}_{\mu(y)} &= \begin{cases} 0.125y - 1 & 8 < y < 16 \\ 1 & 16 \leq y \end{cases} \end{aligned}$$

- 1) Sketch the corresponding Fuzzy Sets for input and output variables. [2]
- 2) Develop appropriate rule base and state corresponding rules (use AND function for rule evaluation). [3]
- 3) Compute the control values  $\eta$  for the input values of  $(\theta, d\theta) = (-1.5, 2)$ . Show all the four stages of the computation: fuzzification, inference, composition and defuzzification. [7]

Question 05

- a) Number of man-hours and the corresponding productivity (in units) are furnished in figure Q5. Fit a simple linear regression equation  $y = mx + c$  applying the method of least squares (LSM). [10]

Man-hours	3.6	4.8	7.2	6.9	10.7	6.1	7.9	9.5	5.4
Productivity (in units)	9.3	10.2	11.5	12	18.6	13.2	10.8	22.7	12.7

Figure Q5

- b) Find the optimal trajectory  $\dot{x}(t)$  corresponding to the minimum of the cost function;

$$J = \int_1^2 [x^2(t) + 2\dot{x}^2(t)] dt$$

With the boundary conditions:  $x(0) = 0, x(2) = 4$  [10]

Question 06

- a) Which control method would you recommend for each of the following applications?
- 1) Servo control with a permanent-magnet DC motor (linear).
  - 2) Active control of a vehicle suspension system (linear, multivariable).
  - 3) Control of a self-diving automated vehicle (nonlinear, complex, difficult to model). [3]
- b) Briefly explain **Linear Quadratic Regulator (LQR)** controller. [3]
- c) Briefly explain **State Feedback Controller**. [2]
- d) What are the available approaches to Improve the accuracy of a neural network. [2]
- e) Explain the application of neural network for robot arm dynamics shows in figure Q6. [10]

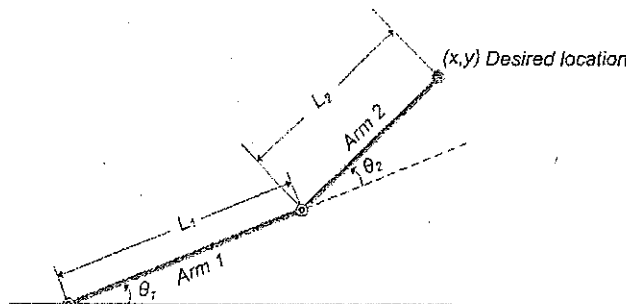


Figure Q6

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