The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Electrical & Computer Engineering

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Study Programme	: Bachelor of Technology Honours in Engineering
Name of the Examination	: Final Examination
Course Code and Title	**: EEX7434 Digital Signal Processing**
Academic Year	: 2021/2022
Date	: 10th February 2023
Time	: 1400-1700 hrs
Duration	**: 3 hours**

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **Seven (7)** questions in **Five (5)** pages.
3. Answer **Any Five (5)** questions. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Relevant additional information is provided at the end of the paper.
6. This is a Closed Book Test **(CBT).**
7. Answers should be in clear hand writing.
8. Do not use red colour pens.

Q1. (a) Consider the signal $x(t) = 4\cos(100\pi t)\cos(200\pi t)$. If the signal $x(t)$ is sampled with a sampling frequency f_s Hz, express the condition that should be satisfied by the sampling frequency f_s in order to ensure the perfect recovery of $x(t)$ from the sampled signal. (Hint: You may use the trigonometric identity $2\cos(\theta)\cos(\phi) = \cos(\theta+\phi) + \cos(\theta-\phi)$.) [4]

(b) Consider the two signals $x_1(t) = 2\cos(40\pi t)$ and $x_2(t) = 2\cos(70\pi t)$.

i. If the signals $x_1(t)$ and $x_2(t)$ are sampled with 100π rad/s, find the digital angular frequencies (measured in rad/sample) of the corresponding discrete-time signals $x_1[n]$ and $x_2[n]$. [4]

ii. Consider the signal $x[n]$ defined by $x[n] = x_1[n] + x_2[n]$. Sketch the frequency spectrum $X(e^{j\omega})$ of $x[n]$ in the range $-3\pi \le \omega \le 3\pi$, where ω is the digital angular frequency. [8]

iii. Assume that the signal $x[n]$ is applied to an ideal reconstruction filter of which the output is $\hat{x}(t) = A_1\cos(\Omega_1 t) + A_2\cos(\Omega_2 t)$, where A_1, A_2, Ω_1, and Ω_2 are constants. What are the values of analog angular frequencies Ω_1 and Ω_2? [4]

Q2. (a) Briefly explain the linearity and the time-invariance properties of a system. [3]

(b) State whether the systems described by the following difference equations are linear and time-invariant. [4]

i. $y[n] = 3x[n+1] + 2x[n] + x[n-1]$

ii. $y[n] = nx[n] + x[n-4]$

(c) State whether systems described by the following difference equations are causal. [3]

i. $y[n] = 3x[n] + 2x[n-1] + x[n-2]$

ii. $y[n] = x[3n] + x[n-4]$

iii. $y[n] = x[n^2] + x[n-2]$

(d) Consider an LTI system with the impulse response $h[n]$ and the input signal $x[n]$ as shown in Figure Q2(d). Find the output signal $y[n]$ for $\forall n \in \mathbb{Z}$. [10]

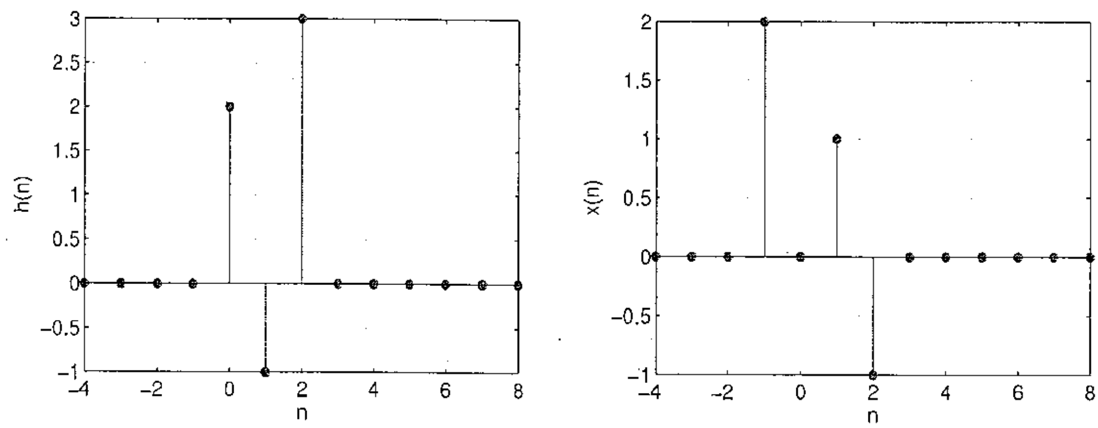

Figure Q2(d): The impulse response and the input signal for Q2(d).

Q3. (a) Consider the signal $x[n] = a^n u[n]$, where a is a constant.

i. Find the z-transform, with the region of convergence (ROC), of the signal $x[n]$ using the first principles. [4]

ii. Sketch the ROC in the complex plane. [1]

(b) Using the answer obtained for **Q3(a)** and the relevant properties of the z-transform, find the z-transform (with the ROC) of the signal [5]

$$y[n] = \left(\frac{1}{2}\right)^n u[n] + \left(\frac{-1}{3}\right)^n u[n].$$

(c) Consider an LTI system having the transfer function

$$H(z) = \frac{1}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{4}z^{-1}\right)}, \qquad |z| > \frac{1}{2}.$$

Using the answer obtained for **Q3(a)** and the relevant properties of the z-transform, find the output signal $y[n]$ of the system if the input signal is $u[n]$. [10]

Q4. Consider the signal $x[n] = a^n u[n]$, where a is a real-valued constant and $u[n]$ is the discrete-time unit-step function.

(a) For the case $|a| < 1$, derive the discrete-time Fourier transform (DTFT) $X(e^{j\omega})$ of $x[n]$ using the first principles. [5]

(b) Using the answer obtained for **Q4(a)**, determine $|X(e^{j\omega})|$ and $\angle X(e^{j\omega})$. [5]

(c) For the case $|a| \geq 1$, can the DTFT $X(e^{j\omega})$ of $x[n]$ be derived? Justify your answer. [4]

(d) Another signal $y[n]$ is defined as $y[n] = a^{|n|}$. Express the DTFT $Y(e^{j\omega})$ in terms of $X(e^{j\omega})$. [6]

Q5. (a) Compute the discrete Fourier transform (DFT) of each of the following discrete-time signals having a finite-length, i.e., $0 \leq n \leq (N-1)$, where N is even:

i. $x[n] = \delta[n - n_0]$, where n_0 is a constant, and $0 \leq n_0 \leq (N-1)$,

ii. $x[n] = a^n$,

iii. $y[n] = e^{j\omega_0 n}$, where ω_0 is a constant, and $-\pi \leq \omega_0 < \pi$. [12]

(b) A discrete-time system is characterized by the transfer function

$$H(z) = \frac{z^4}{4z^4 - 2z^3 + 3z^2 - z + 2}.$$

Determine the stability of the system using the Jury-Marden criterion. [8]

Q6. (a) The coefficients of an Nth order (length $(N+1)$, where N is even) FIR filter $H(z)$ is denoted as $h[n]$, $-N/2 \le n \le N/2$. Express the condition that should be satisfied by the coefficients $h[n]$ of the FIR filter in order to have a zero-phase response. [3]

(b) The ideal frequency response of a zero-phase lowpass filter $H(z)$ is specified as

$$H_I(e^{j\omega}) = \begin{cases} 1, & \text{for } |\omega| \le \omega_c \\ 0, & \text{for } \omega_c < |\omega| \le \pi, \end{cases}$$

where $0 < \omega_c < \pi$ is the cutoff frequency of the lowpass filter. Derive a closed-form expression for the infinite-extent ideal impulse response $h_I[n]$ using the first principles. [7]

(c) A finite-extent impulse response $h[n]$ of length $(N+1)$ can be obtained by multiplying $h_I[n]$ with an appropriate window function $w[n]$ of length $(N+1)$. Obtain the finite-extent impulse response $h[n]$ (or the coefficients) of a 6th order zero-phase FIR lowpass filter having a cutoff frequency 0.3π rad/sample. Use the Hamming window as the window function. The Hamming window of length $(N+1)$ is defined as

$$w[n] = \begin{cases} 0.54 + 0.46\cos\left(\frac{2\pi n}{N}\right), & \text{for } |n| \le \frac{N}{2} \\ 0, & \text{otherwise.} \end{cases}$$

Provide your answer in a table having columns for $h_I[n]$, $w[n]$ and $h[n]$ for required n. [7]

(d) Note that a zero-phase FIR filter is noncausal. How is such a filter converted to a causal filter without changing the magnitude response of the filter. [3]

Q7. A continuous-time elliptic lowpass filter $H_c(s)$ having the transfer function

$$H_c(s) = \frac{0.07(s^2 + 2.58)}{(s + 0.38)(s^2 + 0.31s + 0.51)}$$

is employed to design a discrete-time IIR lowpass filter $H(z)$ using the bilinear transform method. The passband edge Ω_p of $H_c(s)$ is 0.71 rad/s, and the sampling frequency is 10π rad/s. Furthermore, $H(z)$ is realized as a *cascade structure* of a first-order section and a second-order section.

(a) Derive the transfer function $H(z)$. [10]

(b) Determine the passband edge ω_p of $H(z)$. [3]

(c) Draw the realization of $H(z)$ as a cascade structure. Note that the first-order and the second-order sections should be realized using the *direct form II* realizations. [7]

Useful Formulae

- Discrete-Time Fourier Transform (DTFT) of $x[n]$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

- Inverse Discrete-Time Fourier Transform (IDTFT) of $X(e^{j\omega})$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}\,\mathrm{d}\omega$$

- z-Transform of $x[n]$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

- N-point DFT of $x[n]$

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi kn}{N}},\ 0 \le k \le N-1$$

- N-point Inverse DFT (IDFT) of $X[k]$

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j\frac{2\pi kn}{N}},\ 0 \le n \le N-1$$

- Bilinear transform

$$s = \frac{2}{T}\left(\frac{z-1}{z+1}\right),$$

  where T is the sampling period.

- Jury-Marden Criterion: In a stable, linear decrete-time system, all poles are located inside the unit circle centered at the origin.

END OF THE PAPER