THE OPEN UNIVERSITY OF SRI LANKA
Faculty of Engineering Technology
Department of Electrical & Computer Engineering

Bachelor of Technology Honours in Engineering

Final Examination (2016/2017)
ECX6234: Digital Signal Processing

Date: 20th November 2017 (Monday) **Time: 9:30 am – 12:30 pm**

READ CAREFULLY:

Answer **FIVE** questions. Select **at least three** questions from **Section B** (Note: Section B has a total of five questions). All questions carry equal marks.

NOTE: A z-transform table, and some useful hard-to-remember formulas are provided in the **appendix** at the end of this question paper.

SECTION A: *(answer* ***not more than*** *two questions from this section)*

Q1.

(a) A discrete time signal x[n] is shown in Figure Q1. Show graphically the transformed signal given by (-3x[2-n] + 1).(u[n+1] – u[n-4]). Be neat. (16 marks)

(b) Perform the following. Show necessary steps or provide explanations:

(i) Determine whether the system represented by the input/output relationship y[n] = 3x[n] – 4x[n-1] is a memory system or not. (12 marks)

(ii) Determine whether the system represented by the input/output relationship y[n] = 6x[n] + 3 is a linear system or not. (12 marks)

(iii) Determine whether the system represented by the input/output relationship y[n] = median{ x[n-1], x[n], x[n+1] } is a time/shift invariant system or not. (12 marks)

(iv) Determine whether the system represented by y[n] = x[|n|] is a causal system or not. (12 marks)

(c) What can you say about the impulse response h[n] of a linear shift/time-invariant system,

(i) if the system is causal (5 marks)
(ii) if the system is BIBO stable (5 marks)

(d) Showing necessary steps or providing explanations, determine if the LSI system whose impulse response is given by $h[n] = (3/2)^n u[n]$, is a

(i) causal system or not? (13 marks)
(ii) BIBO stable system or not? (13 marks)

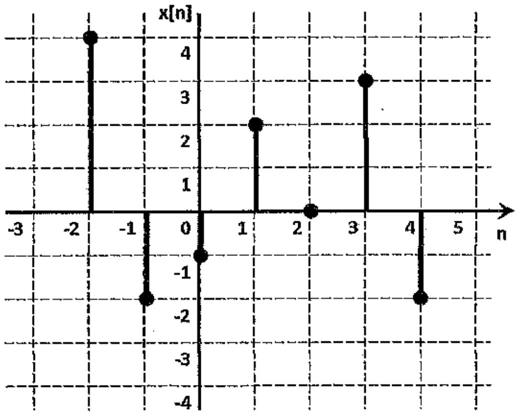
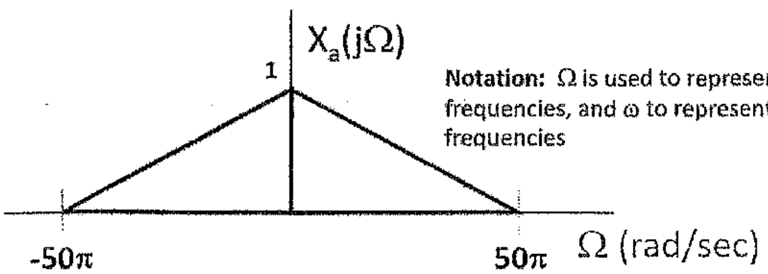


Figure Q1

Q2.

(a) State clearly the sampling theorem and all the required conditions for its applicability. What is meant by Nyquist rate? (10 marks)

(b) An analog signal $x_a(t)$ has the Fourier Transform shown below (Figure Q2-1).



Notation: Ω is used to represent analog frequencies, and ω to represent digital frequencies

Figure Q2-1

(i) How fast must $x_a(t)$ be sampled in order to prevent information from being lost in the sampling process? (15 marks)

(ii) Carefully plot the Fourier transform of the sampled signal for the case where the sampling rate marginally satisfies the minimum criterion determined in part b(i). (25 marks)

(iii) Assuming at least the minimum sampling rate determined above in part b(i), give an expression for reconstructing $x_a(t)$ from its sampled values. (15 marks)

(c) An analog signal $x_a(t)$ is sampled at 15 KHz sampling rate. The Fourier Transform of the resulting sequence of values is plotted in Figure Q2-2.

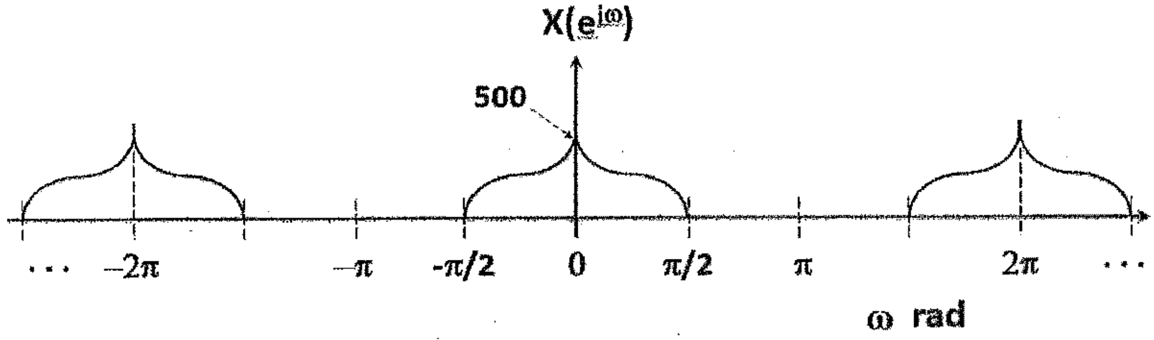


Figure Q2-2

(i) Carefully sketch the Fourier Transform $X_a(j\Omega)$ of $x_a(t)$. (25 marks)

(ii) In theory, would it be possible to reconstruct $x_a(t)$ from its samples? Briefly explain why or why not it is possible. (10 marks)

<u>Q3.</u>

(a) Briefly but clearly explain what is meant by the Region of Convergence (ROC) of the z-transform of a discrete-time signal. (10 marks)

(b) Find the inverse z-transform for the following using the partial-fraction method (a table of useful z-transforms are provided at the end). (30 x 3 = 90 marks):

(i) $$X(z) = \frac{z}{(z-1)(z-2)}, \quad |z| < 1$$

(ii) $X(z)=\frac{z}{(z-1)(z-2)},\quad 1<|z|<2$

(iii) $X(z)=\frac{z}{(z-1)(z-2)},\quad |z|>2$

SECTION B: *(answer **at least** three questions from this section)*

Q4.

We want to design a low-pass IIR digital filter with a passband magnitude characteristic that is constant to within 1 dB for frequencies below $\omega = 0.2\pi$, and stopband attenuation that is greater than 15 dB for frequencies between $\omega = 0.3\pi$ and π. We want to perform the design starting with an analog Chebyshev filter and thereafter use the bilinear transformation method to obtain the required digital filter.

We want to determine the transfer function H(z) for the lowest order Chebyshev design which meets this specification.

(a) Write the necessary equations (inequalities) that mathematically specify the required conditions (pass band and stopband conditions) of the desired digital filter. (15 marks)

(b) From the above, determine the required value of the ripple parameter ε for the intermediate analog Chebyshev filter? (5 marks)

(c) What is the value of Ω_c (the cut-off frequency) for this analog Chebyshev filter? (5 marks).

(d) Determine the required value for the filter order N for this analog Chebyshev filter. A trail and error computation may be required. (15 marks)

(e) Sketch the gain function for the analog filter using the filter order N that you had obtained. Mark all key values clearly. (10 marks)

(f) What is the filter's zero-frequency gain? (4 marks)

(g) Determine the location of the poles of this analog Chebyshev filter. (15 marks)

(h) Determine the s-domain transfer function H(s) for this intermediate analog Chebyshev filter (based on its poles) – make sure it is appropriately normalized? (15 marks)

(i) How is the transfer function H(z) of the required digital filter obtained from this intermediate Chebyshev transfer function? Only explain how through mathematical expressions or statements, but do not simplify any equations or expressions. (8 marks)

(j) Carefully describe how this low-pass (LP) digital filter, $H_{LP}(z)$, can be transformed to a high-pass (HP) digital filter, $H_{HP}(z)$, whose passband is over the range $0.6\pi \leq |\omega| \leq \pi$. (8 marks)

Q5.

(a)

(i) Draw the block-diagram (Direct Form I) realization of the difference equation given as follows (10 marks):

$$y[n] = 0.5x[n] + 0.4x[n-1] - 0.2x[n-2] - 0.3y[n-1] + 0.2y[n-2] - 0.4y[n-3]$$

(ii) Is this a recursive difference-equation or a non-recursive difference equation? Why? (5 marks)

(iii) What is the transfer function H(z) represented by the above difference equation. (10 marks)

(iv) Draw the direct form II block-diagram realization for the above given difference equation? (15 marks)

(v) Draw the transpose direct form I block-diagram realization of the given difference equation. (25 marks)

(vi) Draw the transpose direct form II block-diagram realization of the given difference equation. (25 marks)

(b)

For the block-diagram realization of digital filters shown below (Figure Q5), determine its difference equation representation in a simplified form. Note: Unmarked multipliers can be assumed to be of multiplier value 1.0. (10 marks).

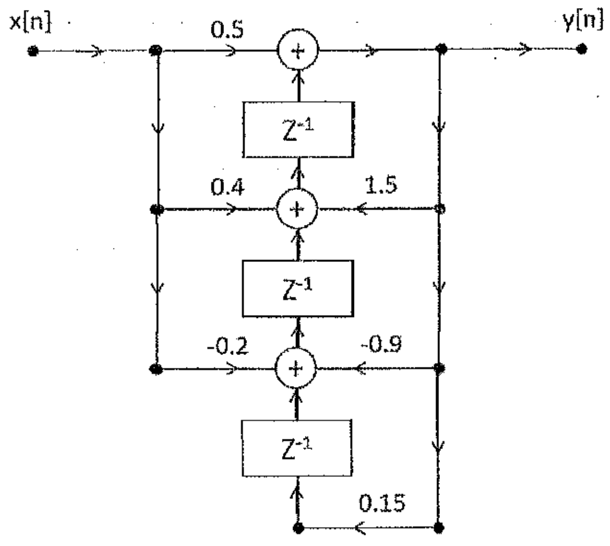


Figure Q5

Q6.

Two discrete-time signals are given by x[n] = {2, -1, 3, 0, 1} and h[n] = {1, -1, 2, 3, -2}.

(a) Using any method of your choice, obtain the **linear convolution** of the above two signals x[n] and h[n] (show, explain, or make self-explanatory each step in whatever method you use). (20 marks)

(b)

(i) Obtain the circular **reflection** of the signal h[n]? (12 marks)

(ii) What is the result of circularly shifting the circularly reflected h[n] obtained above in Q6 b(i) to the right by two samples? (14 marks)

(iii) Obtain the 5-point (five-point) **circular convolution** of the signals x[n] and h[n] (show all steps clearly). Use any convenient method (graphical or using vectors/matrices), but be clear with your steps. (26 marks)

(c) What augmentations should be done to the (original) signals x[n] and h[n] so that the result of circularly convolving these augmented signals is the same as the linear convolution of the (original) signals x[n] and h[n]? Note: you do not need to do the circular convolution in this case, only explain what should be done. (8 marks)

(d) Outline clearly (you do not need to perform the calculations, only outline clearly) how a Discrete Fourier Transform (DFT) based method could be (manually) used in order to obtain the linearly convolving result the (original) signals x[n] and h[n]. (20 marks)

Q7.

(a)

(i) Define mathematically what is meant by the Discrete Fourier Transform (DFT) of a signal in terms of (1) complex exponentials, (2) the usual W_N notation (also include the definition of what W_N represents), and (3) well-defined matrix notation (be clear, and precise). (8 marks)

(ii) Define mathematically what is meant by the Inverse Discrete Fourier Transform (IDFT) of a signal in terms of (1) complex exponentials, (2) the usual W_N notation (also include the definition of what W_N represents), and (3) well-defined matrix notation (be clear, and precise). (8 marks)

(b)

(i) Consider any 4-point discrete-time signal. Determine the values for (usual notation) (1) W_4^0, W_4^1, W_4^2, W_4^3, and W_4^4. (2) What is the value of W_4^7? (10 marks)

(ii) Determine (write) the "$\mathbf{W}_N$ matrix" (used in the N-point matrix representation of the DFT of an N-point signal) for an N = 4 point signal. (12 marks)

(iii) Determine the "inverse $\mathbf{W}_N$ matrix" for the above (Q7 b(ii)) $\mathbf{W}_N$ matrix (N = 4). Note: you do not have to perform a matrix inversion to determine this. (12 marks)

(iv) Find the DFT, X[K], of the discrete–time signal x[n] = {0, 1, 2, 3}. (25 marks)

(v) Find the IDFT of the DFT coefficients X[K] expressed as X[K] = [6, 2+j2, -2, -2-j2]. (25 marks)

Q8.

(a) Write the equation of an N-point DFT, X(k), for an N-point sequence x[n]. (6 marks)

(b) We wish to determine the computational complexity involved in the **direct computation** of a Discrete Fourier Transform (DFT),

(i) What is the number of (1) complex multiplications, and (2) complex additions, required in the above determination for an N-point sequence x[n]? (8 + 8 = 16 marks)

(ii) What would the number of (1) complex multiplications, and (2) complex additions be in this direct DFT-based determination if N = 1024? (8 marks)

(c) How many computational (decomposition) stages are there in the computation of a 1024-point FFT? (6 marks)

(d) Draw a neat and well-labeled flow diagram of an 8-point decimation-in-time FFT algorithm (after all decimations and simplifications are performed). (40 marks)

(e) We wish to determine the computational complexity involved in the decimation-in-time **Fast Fourier Transform (FFT)** algorithm.

(i) What is the number of (1) complex multiplications, and (2) complex additions, required in the above FFT-based determination for an N-point sequence x[n] where N is some integer power of 2? (8 + 8 = 16 marks)

(ii) What would the number of (1) complex multiplications and (2) complex additions be in this FFT-based determination if N = 1024? (8 marks)

APPENDIX

Z-Transform and Inverse Z-Transform Table:

Some Common z-Transform Pairs		
$x[n]$	$X(z)$	ROC
$\delta[n]$	1	All z
$u[n]$	$\frac{z}{z-1}$	$\|z\| > 1$
$-u[-n-1]$	$\frac{z}{z-1}$	$\|z\| < 1$
$\delta[n-m]$	z^{-m}	All z except 0 if $(m > 0)$ or ∞ if $(m < 0)$
$a^n u[n]$	$\frac{z}{z-a}$	$\|z\| > \|a\|$
$-a^n u[-n-1]$	$\frac{z}{z-a}$	$\|z < \|a\|$

Table. Table of Common Z-Transforms

Transformation from a Low-Pass Digital Filter, Prototype of Cutoff Frequency θ_p

Filter Type	*Transformation*	*Associated Design Formulas*
Lowpass	$z^{-1} = \dfrac{Z^{-1} - \alpha}{1 - \alpha Z^{-1}}$	$\alpha = \dfrac{\sin\left(\dfrac{\theta_p - \omega_p}{2}\right)}{\sin\left(\dfrac{\theta_p + \omega_p}{2}\right)}$ ω_p = desired cutoff frequency
Highpass	$-\dfrac{Z^{-1} + \alpha}{1 + \alpha Z^{-1}}$	$\alpha = -\dfrac{\cos\left(\dfrac{\omega_p + \theta_p}{2}\right)}{\cos\left(\dfrac{\omega_p - \theta_p}{2}\right)}$ ω_p = desired cutoff frequency
Bandpass	$-\dfrac{Z^{-2} - \dfrac{2\alpha k}{k+1} Z^{-1} + \dfrac{k-1}{k+1}}{\dfrac{k-1}{k+1} Z^{-2} - \dfrac{2\alpha k}{k+1} Z^{-1} + 1}$	$\alpha = \dfrac{\cos\left(\dfrac{\omega_2 + \omega_1}{2}\right)}{\cos\left(\dfrac{\omega_2 - \omega_1}{2}\right)}$ $k = \cot\left(\dfrac{\omega_2 - \omega_1}{2}\right) \tan\dfrac{\theta_p}{2}$ ω_2, ω_1 = desired upper and lower cutoff frequencies
Bandstop	$\dfrac{Z^{-2} - \dfrac{2\alpha}{1+k} Z^{-1} + \dfrac{1-k}{1+k}}{\dfrac{1-k}{1+k} Z^{-2} - \dfrac{2\alpha}{1+k} Z^{-1} + 1}$	$\alpha = \dfrac{\cos\left(\dfrac{\omega_2 + \omega_1}{2}\right)}{\cos\left(\dfrac{\omega_2 - \omega_1}{2}\right)}$ $k = \tan\left(\dfrac{\omega_2 - \omega_1}{2}\right) \tan\dfrac{\theta_p}{2}$ ω_2, ω_1 = desired upper and lower cutoff frequencies

Some Helpful Formulas:

$$X(e^{j\omega}) = \frac{1}{T_s}\sum_{k=-\infty}^{+\infty} X_a\left(j\frac{\omega}{T_s} + j\frac{2\pi k}{T_s}\right) \qquad X(e^{j\omega}) = \frac{1}{T_s}\sum_{k=-\infty}^{+\infty} X_a\left(j\Omega + j\frac{2\pi k}{T_s}\right)$$

$$\omega = \Omega T$$

$$x_a(t) = \sum_{n=-\infty}^{+\infty} x(n)\frac{\sin\,(\pi/T)(t-nT)}{(\pi/T)(t-nT)}$$

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$H_{desired}(\mathbb{Z}^{-1}) = H_{LP}(z^{-1})\Big|_{z^{-1}=G(\mathbb{Z}^{-1})}$$

$G(\mathbb{Z}^{-1})$ from table

$$\alpha = \epsilon^{-1} + \sqrt{1+\epsilon^{-2}}$$

$$a = \frac{1}{2}\left(\alpha^{1/N} - \alpha^{-1/N}\right) \qquad b = \frac{1}{2}\left(\alpha^{1/N} + \alpha^{-1/N}\right)$$

$$s = -a\Omega_C \sin\left(\frac{\pi}{2N} + k\frac{\pi}{N}\right) \pm j\, b\Omega_C \cos\left(\frac{\pi}{2N} + k\frac{\pi}{N}\right)$$

$$k = 0, 1, 2, \ldots$$

if $s = -a \pm jb$ is a pole pair,

=> $s^2 + 2as + (a^2 + b^2)$ is the corresponding transfer function denominator term

$$\Omega = \frac{2}{T}\, tan\left(\frac{\omega}{2}\right)$$

$$|H(j\Omega)|^2 = \frac{1}{1+\left(\frac{\Omega}{\Omega_C}\right)^{2N}} \qquad |H(j\Omega)|^2 = \frac{1}{1+\epsilon^2 V_N^2\left(\frac{\Omega}{\Omega_C}\right)}$$

$$V_N = \begin{cases} \cos[\,N\cos^{-1}(x)\,] & : x \le 1 \\ \cosh[\,N\cosh^{-1}(x)\,] & : x > 1 \end{cases}$$