



Date: 07.09.2024

Time: 09:00 a.m. – 10:00 a.m.

Answer All questions.

1. a) Consider the vectors $\underline{a} = 3\underline{i} + 4\underline{j}$ and $\underline{b} = 2\underline{i} - \underline{j}$.
- (i) Show that any vector that is coplanar with vectors \underline{a} and \underline{b} can be represented as $\underline{r} = \lambda\underline{a} + \mu\underline{b}$ where λ and μ are scalars. If \underline{a} , \underline{b} and $\underline{c} = 5\underline{i} + 2\underline{j}$ are coplanar vectors, determine the values of λ and μ .
- (ii) Determine whether the vectors \underline{a} and \underline{b} are linearly independent.
- b) Show that the angle between the vectors $2\underline{i} - 2\underline{j} + \underline{k}$ and $\underline{i} + 2\underline{j} + 2\underline{k}$ is $\pi/2$.
- c) In triangle ABC , let D and E be points on sides AB and AC respectively, such that $AD = \frac{1}{3}AB$ and $AE = \frac{1}{3}AC$. Prove that the line segment DE is parallel to BC and has one-third of its magnitude.
2. a) Find the equation of the line through the point $(2, -7, -3)$ and parallel to the vector $-\underline{i} + \underline{j} + 2\underline{k}$. Verify that the point $(-2, -3, 5)$ is on the line.
- b) With respect to a fixed origin O the lines l_1 and l_2 are given by the equations
- $$l_1: \underline{r} = 3\underline{i} + \underline{j} - \underline{k} + \lambda(3\underline{i} + 6\underline{j} + 9\underline{k})$$
- $$l_2: \underline{r} = 2\underline{i} + 5\underline{j} + 0\underline{k} + \mu(-2\underline{i} + 2\underline{j} - 2\underline{k})$$
- Show that the lines l_1 and l_2 intersect at the point $(4, 3, 2)$.
- c) Given that \underline{a} and \underline{b} are unit vectors that are perpendicular to each other, find the value of m such that the two vectors $5\underline{a} + 3\underline{b}$ and $m\underline{a} - 2\underline{b}$ are perpendicular to each other.
- d) If $\underline{a} = 2\underline{i} + 2\underline{j} + \underline{k}$ and $\underline{b} = \underline{i} + 3\underline{j} + 2\underline{k}$ find a unit vector whose direction is perpendicular to the directions of both \underline{a} and \underline{b} .