The Open University of Sri Lanka
B.Sc./B.Ed. Degree/Continuing Education Programme – Level 04
Open Book Test (OBT) – 2024/2025
Applied Mathematics
ADU4302/ADE4302- Vector Calculus
Duration :- One Hour

-232

Date :- 30-08-2024 **Time :- 9.00 a.m. – 10.00 a.m.**

Answer All Questions.

1. (a) Find the domain and range of the function $f(x,y) = \frac{1}{\sqrt{x^2+y^2-4}}$.

   (b) Sketch the level curves of the function $f(x,y) = x^2 + y^2 + 2x + 2y$.

2. Evaluate the following limits, if they exist:

   (a) $\lim_{(x,y)\to(0,0)} \frac{x^4-4y^4}{x^2-2y^2}$ (b) $\lim_{(x,y)\to(0,0)} \frac{2xy}{3x^2+y^2}$

3. If $w = f\left(\frac{r-s}{s}\right)$ then show that $r\frac{\partial w}{\partial r} - s\frac{\partial w}{\partial s} = 0$.

4. Let $w(x,y,z) = x^2 + y^2 + z^2$, where $x = \cos t$, $y = \sin t$ and $z = e^t$. Express w as a function of t and find $\frac{dw}{dt}$ directly. Then find $\frac{dw}{dt}$ using the Chain rule.

5. Find the directional derivative of the function $f(x,y) = 5 - 2x^2 - \frac{1}{2}y^2$ at the point $P(3,\ 4)$ in the direction of $\underline{u} = \cos\frac{\pi}{4}\underline{i} + \sin\frac{\pi}{4}\underline{j}$.

6. Find the equation of the tangent plane to the surface $3xyz = x^3 + y^3$ at the point $P\ (1, 2, \frac{3}{2})$.

7. Find the local extrema of the function $f(x,y) = 120x + 120y - xy - x^2 - y^2$ and determine their nature.