The Open University of Sri Lanka

B.Sc. /B.Ed., Continuing Education Degree Programme

Applied Mathematics – Level 05

ADU5302/ADE5302 – Mathematical Methods

No Book Test (NBT) – 2024/2025

DURATION: ONE HOUR

Date: 06.10.2024. **Time: 09:00h – 10:00h**

ANSWER **ALL** QUESTIONS.

1. The Gamma function denoted by $\Gamma(p)$ corresponding to the parameter p is defined by the improper integral $\Gamma(p)=\int_0^\infty e^{-t}t^{p-1}dt, \quad (p>0)$.

   (i) Using the result $\Gamma(p+1)=p\Gamma(p)$, Compute each of the following:

   (a) $\dfrac{\Gamma 3\Gamma(2.5)}{\Gamma(5.5)}$ (b) $\Gamma\left(\sqrt{-13/2}\right)$

   (ii) Prove that $\int_0^\infty e^{-x^4}dx=\frac{1}{4}\Gamma\left(\frac{1}{4}\right)$.

2. The Beta function denoted by $\beta(p,q)$ is defined by $\beta(p,q)=\int_0^1 x^{p-1}(1-x)^{q-1}dx$,

   where $p>0$ and $q>0$ are parameters.

   Evaluate each of the following:

   (i) $\int_0^1 x^3(1-\sqrt{x})^5dx.$ (ii) $\int_0^{\pi/2}\cos^6 x\,dx$.

3. Let $J_p(x)$ be the Bessel function of order p given by the expansion

$$J_p(x) = x^p \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m}}{2^{2m+p}.m!\Gamma(p+m+1)}$$

(i) Express $J_5(x)$ in terms of $J_0(x)$ and $J_1(x)$.

(ii) Show that $J_n(x)$ is an even function when n is even and an odd function when n is odd.

(iii) Evaluate $\int J_3(x)dx$.

(Hint: You may use the following recurrence relations, if necessary, without proof.)

(i) $\frac{d}{dx}\{x^p J_p(x)\} = x^p J_{p-1}(x)$

(ii) $\frac{d}{dx}\{x^{-p} J_p(x)\} = -x^{-p} J_{p+1}(x)$.

(iii) $\frac{d}{dx}\{J_p(x)\} = J_{p-1}(x) - \frac{p}{x} J_p(x)$ or $xJ'_p(x) = xJ_{p-1}(x) - pJ_p(x)$

(iv) $J'_p(x) = \frac{p}{x} J_p(x) - J_{p+1}(x)$

(v) $J'_p(x) = \frac{1}{2}\{J_{p-1}(x) - J_{p+1}(x)\}$

(vi) $J_{p-1}(x) + J_{p+1}(x) = \frac{2p}{x} J_p(x)$