

The Open University of Sri Lanka  
 B.Sc/B.Ed. Degree Programme – Level 04  
 Open Book Test – 2024/2025  
 Pure Mathematics  
 PEU4300 – Real Analysis 1



Duration: - One Hour.

Date: - 07.09.2024

Time: - From 10.30 a.m. to 11.30 a.m.

Answer All Questions

01) (a) Discuss the boundedness of the following sequences  $\langle a_n \rangle$  where  $a_n$  is given by

$$(i) a_n = 1 + \frac{1}{3} + \frac{1}{3^2} + \cdots + \frac{1}{3^n} \quad ; \text{ for each } n \in \mathbb{N}$$

$$(ii) a_n = \frac{1}{n^2} + \frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \cdots + \frac{1}{(2n)^2} \quad ; \text{ for each } n \in \mathbb{N}$$

(b) Prove that  $a_n = \sqrt{n+1} - \sqrt{n}$  is a decreasing sequence.

(02) (a) Using the definition of limit, prove each of the following:

$$(i) \lim_{n \rightarrow \infty} \frac{3n^4 + 1}{n^4 + n^2} = 3$$

$$(ii) \lim_{n \rightarrow \infty} \frac{1}{3^n} = 0.$$

(b) Let  $\langle x_n \rangle$  be a convergent sequence such that  $\lim_{n \rightarrow \infty} x_n = 1$ . Prove that

$$\lim_{n \rightarrow \infty} x_n^k = 1 \text{ for each } k \in \mathbb{N}.$$

Does it follow that  $\lim_{n \rightarrow \infty} x_n^n = 1$ ? Justify your answer.

