The Open University of Sri Lanka
B.Sc Degree Programme
Pure Mathematics - Level 05
PEU5302 Combinatorics
CAT II - No Book Test 2024/2025
Duration: One hour

**Date: 11.10.2024** **Time: 9:00 a.m. - 10:00 a.m.**

INSTRUCTIONS TO CANDIDATES

- There are **TWO** questions in this paper. Answer **ALL** of them.

1. (a) Find the coefficient of $x^3$ in

$$\left(2x^2 + \frac{1}{x}\right)\left(x^4 - x^2 + 1\right)\left(3x^2 + 1\right).$$

(20 marks)

(b) Find the binomial expansion of $\left(3x^2 - \frac{1}{2x}\right)^{10}$. Hence, deduce that no constant term in the expansion of $\left(3x^2 - \frac{1}{2x}\right)^{10}$.

(20 marks)

2. (a) Using the binomial expansion of $(1+x)^n$, show that

$${}^nC_1 + 2 \cdot {}^nC_2 + \cdots + n \cdot {}^nC_n = n \cdot 2^{n-1}.$$

(20 marks)

(b) Find the coefficient of the term $x^3$ in the expansion of $(1 + x - x^2)^6$.

(20 marks)

(c) Using a combinatorial argument, show that

$${}^nC_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r.$$

(20 marks)