00107

THE OPEN UNIVERSITY OF SRI LANKA

Bachelor of Technology (Civil) – Level 5

CEX5233- STRUCTURAL ANALYSIS

FINAL EXAMINATION - 2016/2017

Time Allowed – 3 Hours

Date: 19th November 2016 Time: 09.30 – 12.30 Hrs

This paper consists of **EIGHT (8)** questions. Answer **any FIVE (5)** questions.

All questions carry equal marks

QUESTION 1

(i) Briefly explain what you understand by **Isotropic and Homogeneous** materials.

(3 Marks)

(ii) A stress field (in N/mm^2) of a certain point of loaded concrete member is shown in Figure Q1. Global coordinates are X, Y and Z.

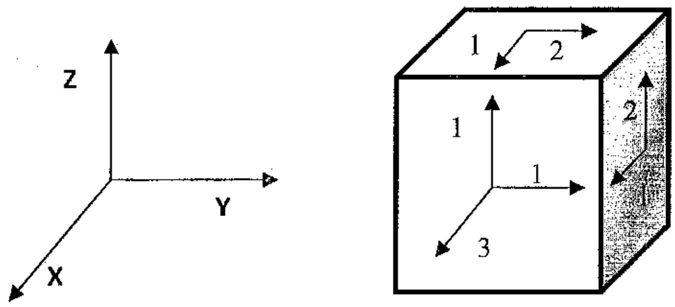

Figure Q1

a) Obtain the stress tensor (2 Marks)

b) Determine the stress invariants (3 Marks)

c) Determine the principal stresses (4 Marks)

d) Obtain the principal axes (4 Marks)

e) If concrete tensile strength is 5 MPa, briefly explain about the safety of the concrete member. (4 Marks)

QUESTION 2

(i) State the difference between "**Plane Stress**" and "**Plane Strain**" problems giving examples. (4 Marks)

(ii) A thin square plate of side 2 units is supported at its bottom corners and is subjected to a pressure loading of intensity P as shown in Figure Q2. The Airy's stress function is proposed to solve this problem as,

$$\emptyset = Ax^2 + Bx^2y + Cy^3 + Dx^2y^3 + Ey^5$$

(a) What is the condition to be satisfied by the constants A, B, C, D and E for $\emptyset$ to be an admissible stress function. (3 Marks)

(b) Find the stress components given by this function. (4 Marks)

(c) Find the constants by using equation in (a) and by imposing following boundary conditions.

Stress boundary conditions on top and bottom edges

Bending moment and axial forces at left and right edges (4 Marks)

(d) Compare this stress field with that from the simple bending theory. (5 Marks)

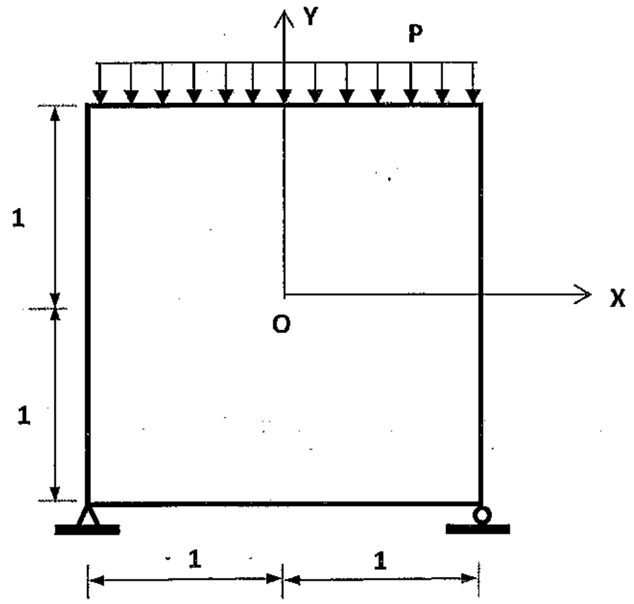

Figure Q2

QUESTION 3

(i) Explain why "**Statically Indeterminate Structures** are more preferred in practice.

(3 Marks)

(ii) A continuous beam (PQRS) is shown in Figure Q3. Flexural rigidities of members PQ and RS are equal to EI and member QR is 2EI. Uniformly distributed load $2w$ is acting on member RS and two concentrated loads $2wl$ and wl in members PQ and QR, respectively.

a) Determine the degree of statical indeterminacy of the beam. (2 Marks)

b) Draw a released structure. (2 Marks)

c) Determine the flexibility matrix for the drawn released structure. (5 Marks)

d) Determine bending moments at Q and R. (6 Marks)

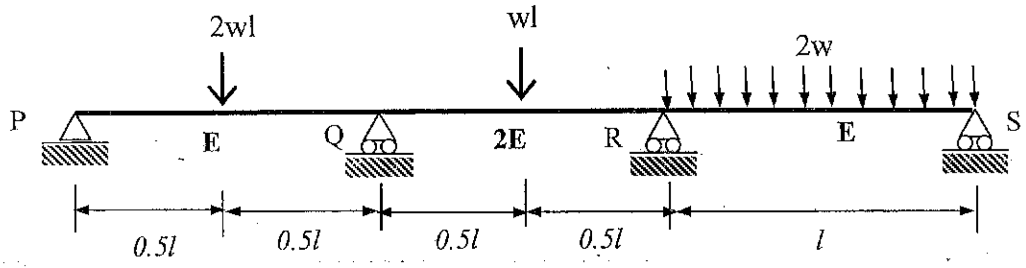

Figure Q3

(iii) If pin support at P is changed to a roller support, briefly explain resulting changes in the beam. (2 Marks)

QUESTION 4

(i) Briefly explain "**Kinematic Indeterminacy**" of a structure. (4 Marks)

(ii) A frame structure is shown in Figure Q4. Flexural rigidities of members are same. Find the free nodal displacements at B using the **displacement method**. You can neglect the axial deformation. (10 Marks)

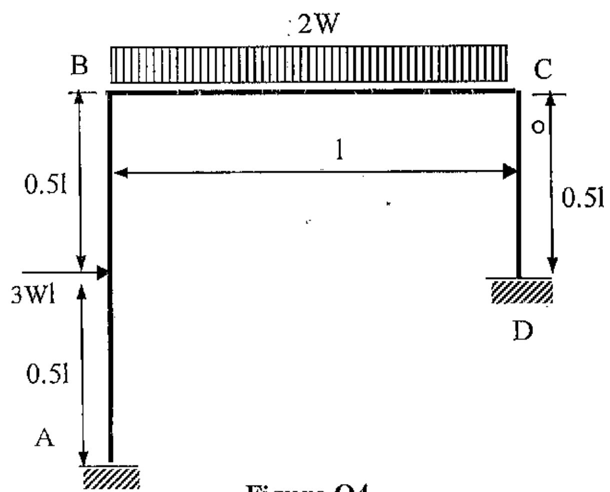

Figure Q4

(iii) Using above results, determine the bending moment at C. (6 Marks)

QUESTION 5

(i) Briefly explain the difference between **Elastic and Plastic Moments of Resistance** of a beam. (3 Marks)

(ii) A two-bay frame structure is shown in Figure Q5. Dimensions and plastic moments of the beam are given in the figure.

(a) Draw possible failure mechanisms. (3 Marks)

(b) Determine load factors for each failure mechanism. (7 Marks)

(c) Determine the most probable failure mechanism. (4 Marks)

(d) Explain how you can ensure the unique solution. (3 Marks)

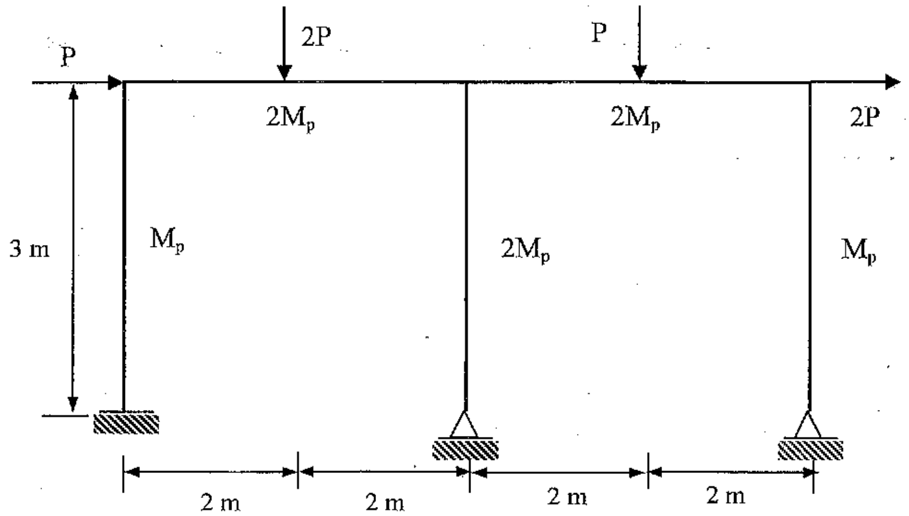

Figure Q5

QUESTION 6

(i) Spherical dome has a radius "R" as shown in Figure Q6. It is subjected to a uniformly distributed load W per meter length on plan.

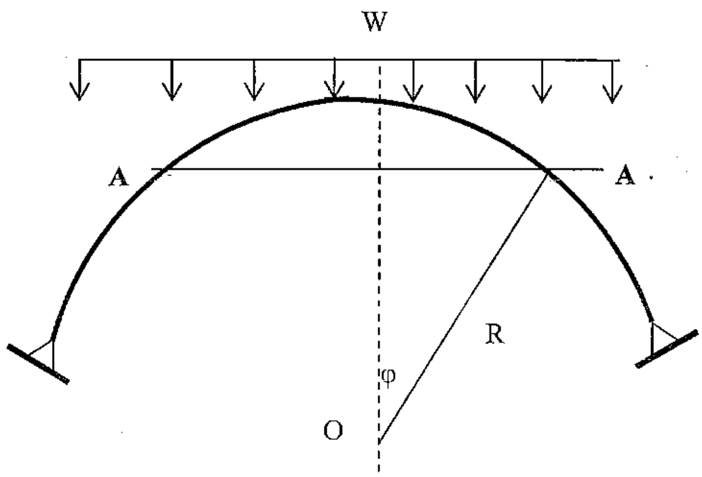

Figure Q6

(a) Determine circumferential and hoop stresses at A-A section level. (10 Marks)

(b) Using the results obtained in section (a), determine the maximum possible value of φ. (4 Marks)

(ii) Briefly following experimental stress measurement techniques.

(a) Photoelasticity method

(b) Grid method

(c) Electrical resistance strain gauge method (6 Marks)

QUESTION 7

(i) What are the assumptions used in the theory of thin plates with small deflection. (4 Marks)

(ii) Governing equation for the uniformly loaded solid circular plate with symmetrical boundary condition is given in polar coordinates as,

$$\nabla^4 w = \frac{1}{r}\frac{d}{dr}\left\{r\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right]\right\} = \frac{q_0}{D}$$

where q_0 is a constant, w is the deflection at a general point.

a) Obtain the general expression for w as

$$w = \frac{q_0 r^4}{64D} + \frac{C_1 r^2}{4}(\log r - 1) + C_2\frac{r^2}{4} + C_3 \log r + C_4$$ (6 Marks)

b) This plate is simply supported at edge ($r = a$), hence obtain the simplified expression for w. (5 Marks)

c) Obtain the maximum deflection of the plate and the its location. (5 Marks)

QUESTION 8

(i) In the plastic theory, **certain conditions** apply when a structure is on the point collapse. Using these conditions how do you obtain,

(a) Lower bound theorem (2 Marks)

(b) Upper bound theorem (2 Marks)

(c) Uniqueness theorem (2 Marks)

(i) Governing equations for axisymmetric shells are given as (14 Marks)

$$\frac{\partial}{\partial S}(rN_S)+\frac{\partial N_{\theta s}}{\partial \theta}-\frac{\partial r}{\partial S}N_\theta+rP_S=0$$

$$\frac{\partial}{\partial S}(rN_{s\theta})+\frac{\partial N_\theta}{\partial \theta}+\frac{\partial r}{\partial S}N_{\theta s}+rP_\theta=0$$

$$r_1N_\theta+r_2N_S+r_1r_2P_Z=0$$

where coordinate system (r, s, ϕ) are as shown in Figure Q8(a).

Using above relationships, find the membrane stress distribution (N_θ and N_S) in a conical water tank supported at the top as shown in Figure Q8(b). You can taker the specific weight of water as γ_w.

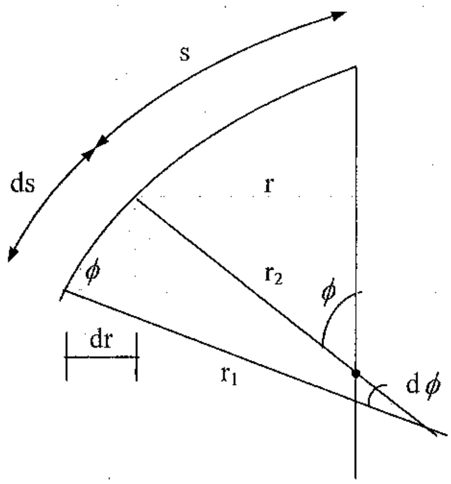

Figure Q8(a)

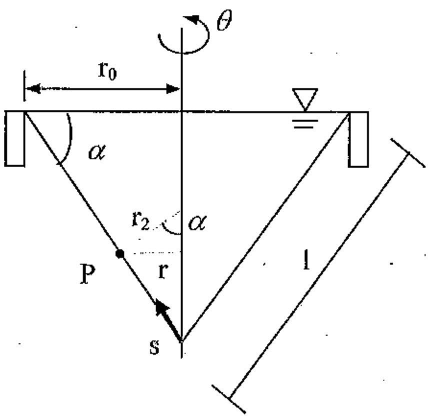

Figure Q8(b)

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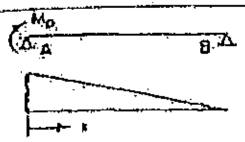
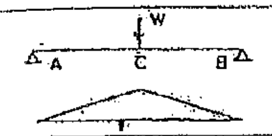
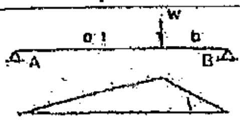
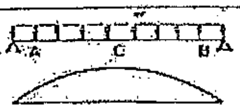
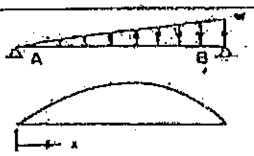
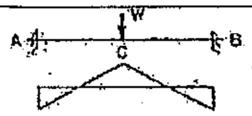
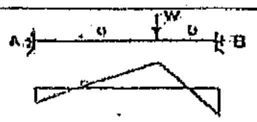
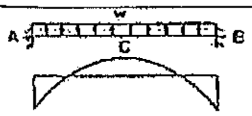
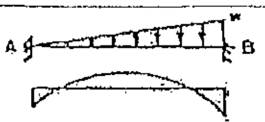
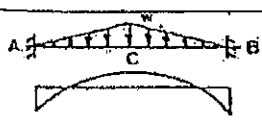
Table 1 Formulas for Beams

Structure	Shear	Moment	Slope	Deflection ↓
Simply supported Beam				
	$S_A = -\frac{M_o}{L}$	M_o	$\theta_A = \frac{M_o L}{3EI}$ $\theta_B = -\frac{M_o L}{6EI}$	$Y_{max} = 0.062\frac{M_o L^2}{EI}$ at $x = 0.422L$
	$S_A = \frac{W}{2}$	$M_o = \frac{WL}{4}$	$\theta_A = -\theta_B = \frac{WL^2}{16EI}$	$Y_c = \frac{WL^3}{48EI}$
	$S_A = \frac{Wb}{L}$ $S_B = \frac{Wa}{L}$	$M_o = \frac{Wab}{L}$	$\theta_A = \frac{Wab}{6EIL}(L+b)$ $\theta_B = -\frac{Wab}{6EIL}(L+a)$	$Y_o = \frac{Wa^2b^2}{3EIL}$
	$S_A = \frac{WL}{2}$	$M_c = \frac{WL^2}{8}$	$\theta_A = -\theta_B = \frac{WL^3}{24EI}$	$Y_c = \frac{5WL^4}{384EI}$
	$S_A = \frac{WL}{6}$ $S_B = \frac{WL}{3}$	$M_{max} = 0.064Wl^2$ at $x = 0.577L$	$\theta_A = \frac{7WL^3}{360EI}$ $\theta_B = -\frac{8WL^3}{360EI}$	$Y_{max} = 0.00652\frac{WL^4}{EI}$ at $x = 0.519L$
	$S_A = \frac{WL}{4}$	$M_c = \frac{WL^2}{12}$	$\theta_A = -\theta_B = \frac{5WL^3}{192EI}$	$Y_c = \frac{WL^4}{120EI}$
Fixed Beams				
	$S_A = \frac{W}{2}$	$M_c = \frac{WL}{8}$	$\theta_A = \theta_B = 0$	$Y_c = \frac{WL^3}{192EI}$
	$S_A = \frac{Wb^2}{L^3}(3a+b)$ $S_B = \frac{Wa^2}{L^3}(3b+a)$	$M_A = -\frac{Wab^2}{L^2}$ $M_B = -\frac{Wba^2}{L^2}$	$\theta_A = \theta_B = 0$	$Y_a = \frac{Wa^3b^3}{3EIL^3}$
	$S_A = \frac{WL}{2}$	$M_A = M_B = -\frac{WL^2}{12}$	$\theta_A = \theta_B = 0$	$Y_c = \frac{WL^4}{384EI}$
	$S_A = \frac{3WL}{20}$ $S_B = -\frac{7WL}{20}$	$M_A = -\frac{WL^2}{30}$ $M_B = -\frac{WL^2}{20}$	$\theta_A = \theta_B = 0$	$Y_{max} = 0.00131\frac{WL^4}{EI}$ at $x = 0.525L$
	$S_A = \frac{WL}{4}$	$M_A = M_B = -\frac{5WL^2}{96}$	$\theta_A = \theta_B = 0$	$Y_c = \frac{0.7WL^4}{384EI}$

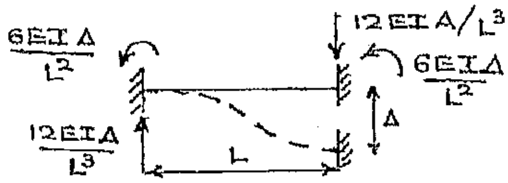
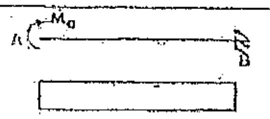
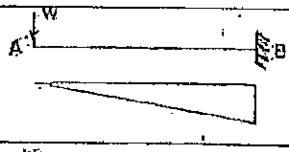
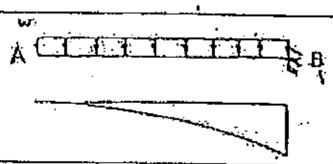
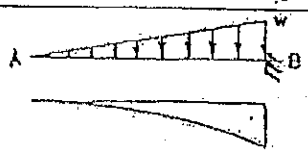
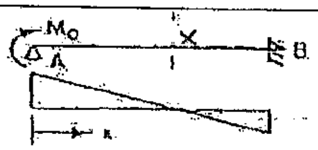
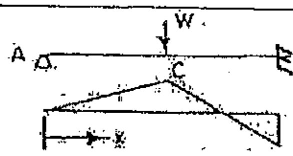
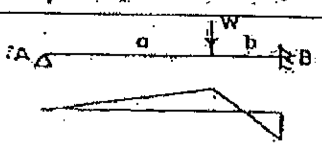
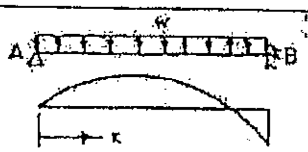
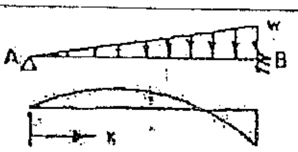


Table 1 Formulas for Beams

Structure	Shear	Moment	Slope	Deflection
Cantilever Beam				
	0	M_o	$\theta_A = \frac{M_o L}{EI}$	$Y_A = \frac{M_o L^2}{2EI}$
	W	$M_B = -WL$	$\theta_A = -\frac{WL^2}{2EI}$	$Y_A = \frac{WL^3}{3EI}$
	$S_B = -WL$	$M_B = -\frac{WL^2}{2}$	$\theta_A = -\frac{WL^3}{6EI}$	$Y_A = \frac{WL^4}{8EI}$
	$S_B = -\frac{WL}{2}$	$M_B = -\frac{WL^2}{6}$	$\theta_A = -\frac{WL^3}{24EI}$	$Y_A = \frac{WL^4}{8EI}$
	$S_B = -\frac{WL}{2}$	$M_B = -\frac{WL^2}{2}$	$\theta_A = -\frac{WL^3}{8EI}$	$Y_A = \frac{11WL^4}{120EI}$
Propped Cantilever				
	$S_A = -\frac{3M_o}{2L}$	$M_B = -\frac{M_o}{2}$	$\theta_A = -\frac{M_o L}{4EI}$	$Y_{max} = \frac{M_o L^2}{27EI}$ at $x = \frac{L}{3}$
	$S_A = -\frac{3M_o}{2L}$	$M_B = -\frac{3WL}{16}$ $M_c = \frac{5WL}{32}$	$\theta_A = \frac{WL^2}{32EI}$	$Y_{max} = 0.00962\frac{WL^3}{ELI}$ at $x = 0.4471$
	$S_A = \frac{Wb^2}{2L^3}(a+2L)$ $S_B = -\frac{Wa}{2L^3}(3L^2 - a^2)$	$M_B = -\frac{Wab}{L^2}(a+\frac{b}{2})$	$\theta_A = \frac{Wab^2}{4EIL}$	$Yo = \frac{Wa^2b^2}{12EIL^3}(3L+$
	$S_A = +\frac{3WL}{8}$	$M_B = -\frac{WL^2}{8}$	$\theta_A = \frac{WL^3}{48EI}$	$Y_{max} = 0.0054\frac{WL^4}{ELI}$ at $x = 0.422$
	$S_A = +\frac{WL}{10}$	$M_{max} = 0.03WL^2$ at $x = 0.447L$ $M_B = -\frac{WL^2}{15}$	$\theta_A = \frac{WL^3}{120EI}$	$Y_{max} = 0.00239\frac{WL}{EI}$ at $x = 0.447$
	$S_A = \frac{11WL}{40}$	$M_{max} = 0.0423WL^2$ at $x = 0.329L$ $M_B = -\frac{7WL^2}{120}$	$\theta_A = \frac{WL^3}{80EI}$	$Y_{max} = 0.00305\frac{WL}{EI}$ at $x = 0.402$