00153

The Open University of Sri Lanka
Department of Electrical and Computer Engineering
Final Examination 2016/2017
ECX5233 – Communication Theory and Systems **CLOSED BOOK**

Time: 0930 – 1230 hrs. Date: 2017-12 -04

Answer any FIVE questions

1.

(a) A periodic signal $x(t)$ has a period of oscillation T_0.

(i) Express the Fourier series of $x(t)$ in complex exponential form and Trigonometric form. [02]

(ii) Simplify the expression given in (i) if $x(t)$ is
(α) an even function ($x(t) = x(-t)$).
(β) an odd function ($x(t) = -x(-t)$). [06]

(iii)

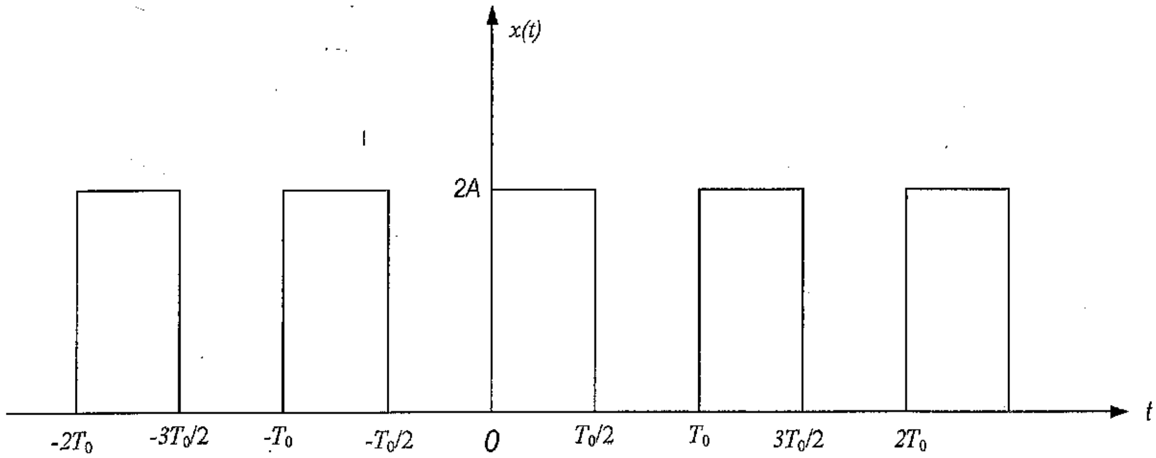

Fig. Q1

(α) Express the signal $x(t)$ shown in Fig.Q1 as complex exponential Fourier Series and trigonometric Fourier Series. [08]

(β) Sketch the amplitude spectrum of $x(t)$. [02]

(γ) Suppose $x(t)$ is transmitted through a low pass filter and the output signal will be $x'(t)$. Draw the amplitude spectrum of $x'(t)$. Comment on your answer. [02]

2.

(a) (i) Define the Fourier transform $Y(\omega)$ of a non periodic signal $y(t)$. [02]

(ii) What information of the signal $y(t)$ can be extracted from $Y(\omega)$? [02]

(b) Find the *Fourier Transforms* of following functions and sketch them. [10]

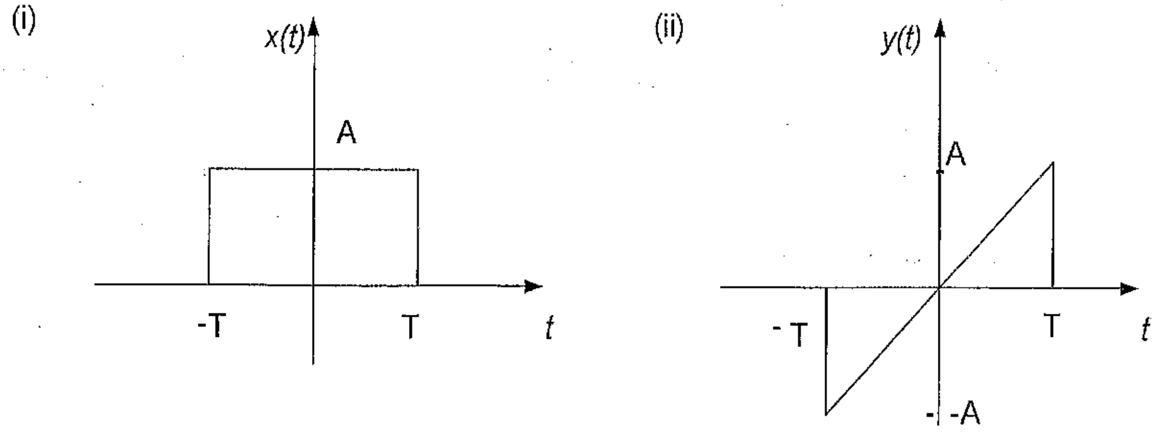

Fig. Q2

(c) The *Convolution* of two waveforms *x(t)* and *y(t)* shown in Fig. Q2 can be defined as follows:

$$s(t) = x(t) * y(t) = \int_{-\infty}^{+\infty} x(\tau).y(t-\tau)d\tau \text{ -Convolution Integral.}$$

Evaluate the convolution of *x(t)* and *y(t)* shown above. [06]

3.

(a) Consider a system in Fig. Q3(a) with output *y(t) = g(t).x(t)* where *x(t)* is the square wave depicted in Fig. Q3(b).

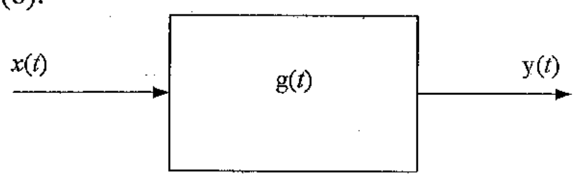

Fig. Q3(a)

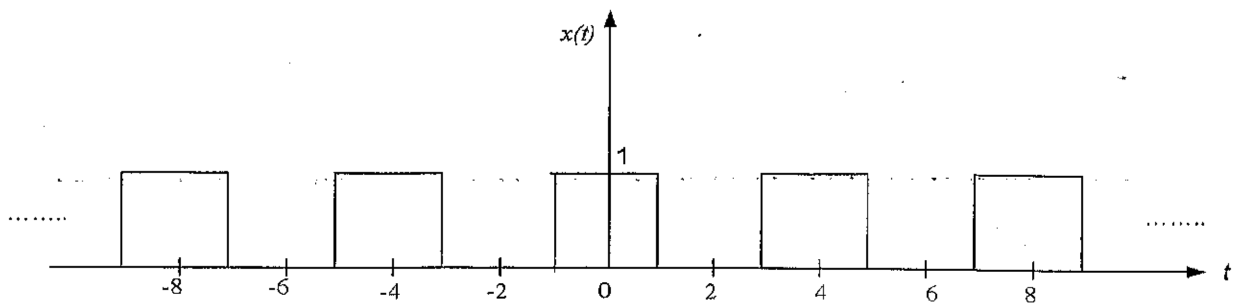

Fig. Q3(b)

(i) Find *Y(ω)* in terms of *G(ω)* where *Y(ω)* and *G(ω)* are Fourier transforms of *y(t)* and *g(t)* respectively. [02]

(ii) Sketch $Y(\omega)$ if $g(t) = cos(t/2)$ [05]

(iii) What is the impact of the channel on the spectral components of $x(t)$? [03]

(b) (i) If an amplitude modulated carrier $A(1+m\cos\omega_m t)\cos\omega_c t$ is used as $x(t)$, derive an expression for $Y(\omega)$. [05]

(ii) If $x(t) = A(1+s(t))\cos\omega_c t$ where $s(t)$ is a non sinusoidal base band signal, derive an expression for $Y(\omega)$ in terms of $S(\omega)$, where $S(\omega)$ is the Fourier Transform of $s(t)$. [05]

4.

(a) A repetitive waveform is given as $s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$ where $T_s = \frac{2\pi}{8\omega_0}$ where $\omega_0 = 2\pi/T$.

(i) Draw $s(t)$ and express it using Fourier Series. [03]

(ii) Find the Fourier transform of $s(t)$. [04]

(b) A signal $x(t) = Sin\omega_0(t)$ is sampled using the above impulse train $s(t)$ and the sampled signal $x_s(t) = x(t).s(t)$.

(i) Sketch $x_s(t)$. [03]

(ii) Find the Fourier transform $X_s(\omega)$ of $x_s(t)$. [04]

(iii) Sketch $X_s(\omega)$ and explain whether $x(t)$ can be recovered without distortion. [04]

(c) If the signal $x(t)$ is sampled using a pulse train $r(t) = \sum_{n=-\infty}^{\infty} p(t - nT_0)$, where $p(t)$ is a rectangular pulse having a width τ and height 1,

ie. $$p(t) = \begin{cases} 1, \text{ for } |t| \leq \tau/2 \\ 0 \quad \text{otherwise} \end{cases}$$

give an expression for the sampled signal $x_s(t)$. [02]

5.

(a) Define *Auto Correlation function* $\Re_{xx}(\tau)$, of a random process $X(t)$. [01]

(b) Define *Cross Correlation function* $\Re_{xy}(\tau)$, of two random processes $X(t)$ and $Y(t)$. [01]

(c) What is understood by two *independent* random processes $X(t)$ and $Y(t)$? [02]

(d) $X(t)$ and $Y(t)$ are two random processes. A random process $Z(t)$ is defined by

$Z(t) = X(t) + Y(t)$.

Derive an expression for the autocorrelation function $\Re_{zz}(\tau)$ of $Z(t)$ in terms $\Re_{xx}(\tau), \Re_{yy}(\tau), \Re_{xy}(\tau)$ and $\Re_{yx}(\tau)$.

where $\Re_{yy}(\tau)$ = *Auto Correlation function* of random process $Y(t)$.

$\Re_{yx}(\tau)$ = *Cross Correlation function* of random processes $Y(t)$ and $X(t)$. [04]

(e) (i) Sketch the ensemble of the random process $x(t) = a\cos(\omega_c t + \theta)$ where ω_c is a constant and a and θ are independent random variables uniformly distributed in the ranges (-1, 1) and $(0, 2\pi)$ respectively. [02]

(ii) Determine $\overline{X(t)}$ and $R_x(t_1, t_2)$. [04]

(iv) Hence determine whether the process is wide sense stationary. [02]

(v) Also determine whether the system is ergodic. [02]

(vi) If the system is wide sense stationary, what is its power P_s? [02]

6.

(a) Consider a source **m** emitting messages $m_1, m_2, \ldots, m_i \ldots m_n$ with probabilities $P_1, P_2, \ldots P_i \ldots, P_n$ respectively.

(i) Write an expression for the information content $I(m_i)$ of the message m_i. [01]
(ii) Write an expression for the average information content $H(m)$. [01]

(b) Consider the binary symmetric channel (BCS) shown in Fig. Q5(a).

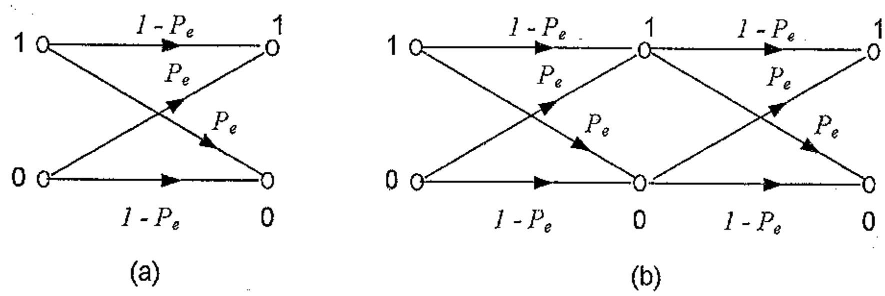

Fig. Q5

(i) Find the channel matrix for the BSC channel in Fig.5(a). [02]

(ii) Fig. Q5(b) shows a cascade of two such channels.
(α) Find the probabilities of P(1|1) and P(1|0) in terms of P_e. [02]
(β) Show that the channel matrix of this cascaded channel is M^2. [03]

(iii) If the two BSC channels above have error probabilities P_1 and P_2 with channel matrices M_1 and M_2 respectively. Show that the channel matrix of the cascade of these two channels is $M_1 M_2$. [03]

(c) A discrete memoryless source has an alphabet of seven symbols whose probabilities of occurance are as given here.

Symbol	S_0	S_1	S_2	S_3	S_4	S_5	S_6
Probability	0.25	0.25	0.125	0.125	0.125	0.0625	0.0625

Find,

(i) the Huffman code for the message. [04]
(ii) average codeword length [02]
(iii) the entropy of the specified discrete memoryless channel [02]

7.
(a)

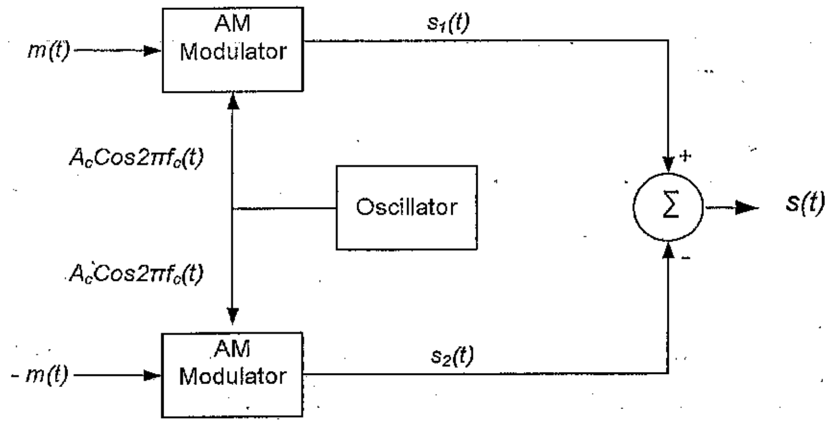

Fig. Q7

Fig. Q7 shows the block diagram of a *balanced modulator*. The input applied to the top AM modulator is *m(t),* whereas and that applied to lower AM modulator is – *m(t)*. Assume that these two modulators have the same amplitude sensitivity.
Show that the output *s(t)* of the balanced modulator consists of a DSB-SC modulated signal. [05]

(b) An angle modulated signal with carrier frequency $\omega_c = 2\pi \times 10^6$ is described by the equation:

$$\varphi(t) = 10\cos(\omega_c t + 5\sin 300t + 10\sin 2000\pi t)$$

(i) Find the power of the modulated signal [02]
(ii) Find the frequency deviation Δf. [02]
(iii) Find the deviation ratio β. [03]
(iv) Find the phase deviation Δϕ. [03]
(v) Estimate the bandwidth $\varphi(t)$. [03]
(vi) What is narrow band *phase modulation*? [02]

8.

(a) A communication system receives a random variable Y which is defined as $Y = X + N$ where X is the input random variable and N is the channel noise N.
X takes on the values $-1/4$ and $1/4$ with $P[X = 1/4] = 0.6$. Let $f_N(n)$ denotes the probability density function of the channel noise and let X and N be independent. The receiver must decide for each received $Y = y$ whether the transmitted X was $-1/4$ or $1/4$.
If N is uniform in $(-1/2, 1/2)$,

(i) Determine and sketch the probability density functions of
- (α) $f_Y(y|X = 1/4)$
- (β) $f_Y(y|X = -1/4)$
- (γ) $f_Y(y)$

[09]

(ii) Also determine the optimal rule such that the probability of correct decision is maximized. [03]

(b) Briefly explain the following terms related to antennas mentioning how they are important in antenna design:
- (i) Antenna gain
- (ii) Radiation pattern
- (iii) Antenna beamwidth
- (iv) Antenna efficiency

[08]