**The Open University of Sri Lanka**
**Department of Mathematics**
**Advanced Certificate in Science Programme**
**MYF2519 - Combined Mathematics I – Level 2**

**Final Examination 2024/2025**

**Date: 12-10-2024** **From 1:30 pm. To 4:30 pm.**

Answer **All** Questions in **Part A** and Answer **Five** Questions in **Part B.**

## PART A

1. Find the domain and range of the function $y = \frac{(3x+1)}{\sqrt{(25-x^2)}}, x \neq \pm 5$ .

2. The functions $f(x)$ and $g(x)$ are defined by $f: x \rightarrow 2x+7;\ x \in \mathbb{R}$, and $g: x \rightarrow x^3 - 1;\ x \in \mathbb{R}$. Find the following:

   (a) $f \circ g$,
   (b) $f^{-1}$,
   (c) $g^{-1}$,
   (d) $f^{-1} \circ g^{-1}$,
   (e) $g^{-1} \circ f^{-1}$.

3. Solve the equation, $4^x - 6 \times 2^x - 16 = 0$.

4. Solve the equation, $16 \log_x 3 = \log_3 x$.

5. Given that the polynomial function $f(x) = x^3 + 2x^2 - 3x + k$, where $k$ is a constant, when divided by $(x-1)$, has remainder 2, find the value of $k$.

6. Given that $a, b$ and $c$ are real numbers. Show that the roots of the equation $(x-a)(x-b) = c^2$ are real.

7. Solve the inequality $\frac{x-3}{x+1} \leq 2$.

8. The line $3x + 2y = 12$ meets the $y$-axis at $A$ and $x$-axis at $B$. Find the area of the triangle $OAB$ where $O$ is the origin.

9. Show that $2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} = \frac{\pi}{4}$.

10. Show that

$$\frac{1}{\operatorname{cosec}\theta - \cot\theta} = \operatorname{cosec}\theta + \cot\theta$$

## PART B

11. (a) Let $f(x) = ax^3 + bx^2 - 11x + 7$, where $a, b \in \mathbb{R}$. If $(x-1)$ is a factor of $f(x)$, and the remainder when $f(x)$ is divided by $(x-2)$ is $-7$,

(i) find the values of $a$ and $b$.
(ii) Find the other quadratic factor of $f(x)$.

(b) Given that

$$x^3 + 11x + 8 = (x^2+9)(x-1) + A(x^2+9) + +B(x-1)^2, \text{ for all } x \in \mathbb{R}.$$

Find the values of $A$ and $B$.

Hence, write down $\frac{x^3+11x+8}{(x-1)^2(x^2+9)}$ in partial fractions.

12. Let $a, b, c \in \mathbb{R}$, $a \neq 0$ and $a - b + c \neq 0$. Let $f(x) = ax^2 + bx + c$. Show that $x = -1$ is not a root of the equation $f(x) = 0$. Let $\alpha$ and $\beta$ are the roots of $f(x) = 0$. Write down $(\alpha + \beta)$ and $\alpha\beta$ in terms of $a, b, c$.

(a) Show that the quadratic equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is $g(x) = cx^2 + bx + a = 0$.

(b) Show that if the roots of the equation $f(x) = 0$ are real, then the roots of $g(x) = 0$ are also real.

13. (a) Sketch the graph of $y = 2|x-1|$ and $y = |x| + 2$ in the same diagram. Hence or otherwise, find all real values of $x$ satisfying the inequality $2|x-1| > |x| + 2$.

(b) Sketch the graph of $y = 1 - 2|x|$ and of $y = \left|x - \frac{1}{2}\right|$, in the same diagram.
Hence, or otherwise find the values of $x$ satisfying the equation $1 - 2|x| = \left|x - \frac{1}{2}\right|$.

14. (a) Write down the Sine rule for any triangle $ABC$.

With the usual notation show that $\tan\frac{(B-C)}{2} = \left(\frac{b-c}{b+c}\right)\cot\frac{A}{2}$.

(b) (i) Show that $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$ and $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$.

(ii) Let $f(x) = \sqrt{3}\sin x - \cos x - 1$. Express $f(x)$ in the form $A\sin(x+\alpha) + B$, where $A(>0)$, $B$ and $\alpha \left(0 < \alpha < \frac{\pi}{2}\right)$ are constants to be determined.
Deduce that $-3 \leq f(x) \leq 1$. Solve the equation $f(x) = 0$.

15. (a) Write down the maximum and minimum values of $y = 2\sin x$, where $0 \leq x \leq 2\pi$.

(b) Sketch the graph of $y = 2\sin x$.

(c) Also sketch the graph of $y = 2\sin x + 1$ in the same diagram.

(d) Find all the roots which satisfy $y = 2$ and $y = 2\sin x$ when $0 \leq x \leq 2\pi$.

(e) Find also all the roots which satisfy $y = 1$ and $y = 2\sin x + 1$ when $0 \leq x \leq 2\pi$.

16. (a) Show that the perpendicular distance from the point $(x_0, y_0)$ to the line $ax + by + c = 0$ is $\frac{[ax_0 + by_0 + c]}{\sqrt{a^2+b^2}}$.

(b) Find the perpendicular distance from the point $(1, 1)$ to the line $3x + 4y + 8 = 0$.

(c) Find the equations of two straight lines which are one unit of perpendicular distance from the origin and parallel to the straight line $3x + 4y + 8 = 0$.

17. (a) Show that the angular bisector of the acute angle between two straight lines $x + y + 4 = 0$ and $7x + y - 8 = 0$ is $2x + y + 2 = 0$.

(b) Show that the point $(2, -6)$ lies on the given two lines. Show also that the point $(0, -2)$ satisfies the angular bisector of the acute angle.

(c) Find the equation of the line which is perpendicular to the bisector of the acute angle through $(0, -2)$.