The Open University of Sri Lanka
Department of Mathematics
Advanced Certificate in Science Programme
MYF2521 - Combined Mathematics 3 - Level 2
Final Examination 2024/2025

Date: 13-10-2024 **From: 09:30 am. To 12:30 pm.**

Answer **All** Questions in **Part A** and Answer **Five** Questions in **Part B.**

PART A

1. Using the **Principle of Mathematical Induction**, prove that $9^n - 4^n$ is divisible by 5 for all positive integers n.

2. How many terms are there in the geometric progression, $3, 6, 12, 24, 48, \cdots\cdots, 3072$?

3. Find the coefficient of x^{41} in the binomial expansion of $\left(2x^3 + \frac{1}{15x}\right)^{15}$.

4. A committee of 8 members is to be form from a group consisting of 6 men and 5 women

   (a) How many committees can be formed.
   (b) How many committees contain 4 men and 4 women.

5. Express the complex number z, $z = 1 + i$ in the modulus and argument (polar) form. Hence find z^8 in cartesian form.

6. If $P = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $Q = \begin{pmatrix} 4 & 2 \end{pmatrix}$, find PQ. If $R = \begin{pmatrix} 1 & 1 \\ 5 & 1 \end{pmatrix}$, then find the matrix A where $A = PQ - R$.

7. Evaluate the following limits.

   (a) $\lim\limits_{x \to \infty} \frac{x^2+x+1}{2x^2-x+3}$ (b) $\lim\limits_{x \to 0} \frac{\sin 7x}{\sin 9x}$

8. Let $x = t + \ln t$ and $y = t - \ln t$, where t is a parameter. Find $\frac{dy}{dx}$ at the point $(1, 1)$.

9. Show that $\int_0^1 e^{4x} dx = \frac{1}{4}(e^4 - 1)$.

10. Let S be the circle defined by $S \equiv x^2 + y^2 - 4x - 2y + 4 = 0$.

    Find the centre and the radius of the circle and show that the point $(1, 1)$ lies on the circle.

PART B

11. Let $U_r = \frac{1}{(r+2)(r+3)}$. Determine the values of the real constants A and B such that, $U_r = \frac{A}{(r+2)} + \frac{B}{(r+3)}$.

Hence, find $f(r)$ such that $U_r = f(r) - f(r+1)$.

Show that $\sum_{r=1}^{n} U_r = \frac{1}{3} - \frac{1}{(n+3)}$.

Deduce that the infinite series $\sum_{r=1}^{\infty} U_r$ is convergent and find its sum.

12. (a) The matrix A is given by $A = \begin{pmatrix} a & -5 \\ 1 & a+4 \end{pmatrix}$. Show that A^{-1} exists for all $a \epsilon \mathbb{R}$.

(b) The matrices $P = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$, $Q = \begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix}$ and $R = \begin{pmatrix} 1 & -7 \\ 0 & 2 \end{pmatrix}$ are such that

$A = PQ + R$. Show that $a = 2$.

(c) Find A^{-1}. Hence, solve the simultaneous equations

$$2x - 5y = 8 \text{ and}$$
$$x + 6y = 13$$

13. (a) Let z, w be complex numbers and let $\bar{z}$ and $\bar{w}$ be the conjugates of z and w respectively.

(i) Show that $|z|^2 = z\bar{z}$, $|z| = |\bar{z}|$ and $z + \bar{z} = 2\text{Re}(z)$.

(ii) $z\bar{w} + w\bar{z} = 2\text{Re}(z\bar{w})$.

Hence, show that $|z+w|^2 = |z|^2 + 2\text{Re}(z\bar{w}) + |w|^2$.

(iii) Similarly write down the expression for $|z-w|^2$. Deduce that

$$|z+w|^2 + |z-w|^2 = 2(|z|^2 + |w|^2).$$

(b) (i) Let $z = \frac{1}{2}(\sqrt{3} + i)$. Write down z in modulus and argument (polar) form. Find $|z|$ and $Arg\, z$.

(ii) Let $w = \frac{1}{2}(\sqrt{3} - i)$. Show that $|z+w|^2 + |z-w|^2 = 4$.

14. (a) Differentiate following functions with respect to x.

(i) $y = (2x+1)^2(x^2+1)^3$

(ii) $y = \sqrt{\frac{\sin x + \cos x}{\sin x - \cos x}}$

(iii) $y = \ln|e^x + e^{-x}|$

(iv) $y = (x^2+1)^{\tan^{-1} x}$

(b) An open cylindrical vessel is to be made from a given amount of uniform thin material. Show that this vessel can be made using a minimum quantity of material when its radius and height are equal.

15. (a) Let $f(x) = \frac{(x-1)(2x-5)}{(x-4)^2}, x \neq 4$. Show that $f'(x)$, the derivative of $f(x)$ is given by $f'(x) = \frac{9(2-x)}{(x-4)^3}$ for $x \neq 4$.

Hence find the intervals on which $f(x)$ is increasing and $f(x)$ is decreasing. Find the turning points, vertical and horizontal asymptotes of $f(x)$ and sketch the graph of $f(x)$.

(b) Find the volume generated when the part of the curve $y^2 = 4(x-1)$ in the range $0 \leq y \leq 2$ is rotated by four right angles about the y-axis.

16. (a) Express $\frac{x-2}{(x-1)(x-4)}$ in partial fractions and hence evaluate $\int \frac{x-2}{(x-1)(x-4)} dx$.

(b) Using integration by parts, show that $\int_0^1 x \tan^{-1} x \, dx = \frac{1}{4}(\pi - 2)$.

(c) Show that the area of the region enclosed by the curves $y = 2x - x^2$, $x = 0$, $x = 1$ and $y = 0$ is $^2/_3$.

17. Consider the circles S_1 and S_2 where

$S_1 \equiv x^2 + y^2 - 4y - 5 = 0$ and

$S_2 \equiv x^2 + y^2 + {^3/_2}x + 3y - 1 = 0$.

Write down the Centre and radius of the circles S_1 and S_2. Show that the point $(2, 0)$ on the circle S_2 .Show both circles intersect orthogonally.

Find the equation of the circle which passes through the origin and the points of intersection of the two circles S_1 and S_2.