The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc. / B. Ed. Degree Programme

Department	**: Mathematics**
Level	**: 03**
Name of the Examination	**: Final Examination**
Course Title and - Code	**: Applied Calculus I – ADE3200**
Academic Year	**: 2024/25**
Date	**: 15.12.2024**
Time	**: 09:30a.m. –11:30a.m.**
Duration	**: Two Hours**

General Instructions

1. Read all instructions carefully before answering the questions.
2. **This paper consists of TWO sections: Section A and Section B in Three (03) pages.**

Section A

- o This section is **compulsory.**
- o It consists of Three questions.

Section B

- o This section consists of FIVE(05) Essay Type Questions and each question carries 100 marks.
- o Answer only any THREE(03) questions of them in a separate answer booklet given by the University .

3. Answer for each question should commence from a new page.
4. Involvement in any activity that is considered as an exam offense will lead to punishment.
5. Use blue or black ink to answer the questions.
6. Clearly state your index number in your answer script.

PART A

1. a) Determine whether the following statements are **TRUE** or **FALSE**. In each case justify your answer.

   i) Let f be a function defined by $f(x)$ in the interval (-1,1). If $f'(x) > 0$ for all x, then f is increasing in the interval (-1,1). (10 marks)

   ii) The sequence $\{(-1)^n\}_{n=0}^{\infty}$ is a bounded sequence. (10 marks)

   iii) If $\lim_{n\to\infty} a_n = 0$, then the series $\sum_{n=1}^{\infty} a_n$ is convergent. (10 marks)

   iv) The domain of $y = \sqrt{9 - x^2}$ is all real numbers greater than or equal to zero. (10 marks)

   b) Show the following:

   i) $\lim_{n\to\infty} \left(\frac{1}{3^n}\right) = 0$ (15 marks)

   ii) $\lim_{n\to\infty} \frac{1}{\sqrt[3]{n}} = 0.$ (15 marks)

   c) Find the derivative, $\frac{dy}{dx}$ of $\sin y + x^2y^3 - \cos x = 2y$ using implicit differentiation. (30 marks)

[Total marks 100]

P.T.O

PART B

Answer only any THREE (03) questions

2. Consider the function f defined by $f(x) = \begin{cases} x^2 - 15 & for -7 < x \leq 0 \\ |x+4| & for\ 0 < x < 6 \\ 3x & for\ \ 6 \leq x < 8 \end{cases}$

a) Find the domain and range of f. (20 marks)

b) Solve $f(x) = 0$. (10 marks)

c) Find $\lim_{x \to 0^+} f(x)$ and $\lim_{x \to 0^-} f(x)$. (20 marks)

d) Sketch the graph of f. (50 marks)

[Total marks 100]

3. Answer to the following questions:

a) If $g(x) = 1 - x^3$, find $g'(0)$ and use it to find an equation of the tangent line to the curve $y = 1 - x^3$ at the point $(0, 1)$.

(25 marks)

b) Using suitable rules of derivatives, find $g'(x)$, where $g(x) = \sqrt{1 + \sqrt{1 + \sqrt{x}}}$.

(35 marks)

c) Show that the derivative of the function f defined by $f(x) = \sqrt{x}$, exists in the interval $(0, \infty)$.

Hence, show that $f(x)$ is not a differentiable function. (40 marks)

[Total marks 100]

P.T.O

4. a) A water tank is in the shape of a right circular cone with axis vertical and vertex downward. The tank is of radius 3 m and height of 5 m. Initially, the tank is completely filled with water, but at time $t = 0$ (in seconds), a small hole at the vertex is opened and the water begins to drain. When the height of water in the tank has dropped to 3 m, the water is flowing out at the rate of $2\ m^3/s$. At what rate, in meters per second, is the water level dropping then?

(**Note**: The volume of a right circular cone is $\frac{1}{3}\pi r^2 h$, where r is the radius and h is the height)

(60 marks)

b) Consider the following function f defined by:

$$f(x) = \begin{cases} x^2 - a^2 x & for\ x < 2 \\ 4 - 2x^2 & for\ x \geq 2 \end{cases}$$

Find all values of a that makes f continuous at $x = 2$. Using the definition of continuity, justify your answer.

(40 marks)

[Total marks 100]

5. a) State the formal definition of the limit of a sequence a_n as n approaches to infinity. (10 marks)

b) i) Determine whether the sequence $\left\{\frac{n}{n+1}\right\}_{n=0}^{\infty}$ converges or diverges. If it converges, find the limit.

(40 marks)

ii) Determine whether the sequence $\left\{3^{-n}\right\}_{n=0}^{\infty}$ is increasing, decreasing, bounded below, bounded above and/or bounded. (50 marks)

[Total marks 100]

6. a) Use comparison test to show that the series $\sum_{n=1}^{\infty} \frac{5}{2n^2+4n+3}$ is convergent. (50 marks)

b) Determine whether each of the following series converges or diverges.

i) $\sum_{n=1}^{\infty} \frac{n!}{\pi^n}$ (20 marks)

ii) $\sum_{n=1}^{\infty} \left(\frac{3+2n}{1+5n}\right)^n$. (30 marks)

[Total marks 100]

********* END OF THE QUESTION PAPER *********

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